日本物理学会　2012秋季大会(横浜国立大学)
平成24年9月20日(火)10:45～11:15

コンプレックスプラズマにおける微粒子構造形成

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コンプレックスプラズマにおける微粒子構造形成

１　歴史的な背景：宇宙塵，ダスト,微粒子

２　コンプレックスプラズマの特徴

　実験室におけるコンプレックスプラズマ

　ダストとプラズマの相互作用

3　微粒子によるバウショック形成（実験）

4　鎖形成、螺旋構造　（シミュレーション）

5　微粒子と電荷の相互作用，　分極効果（理論）

6　1次元、２次元格子と集団運動（理論）

　結び

END

Dawn of the Physics of Complex Plasmas

Irving Langmuir

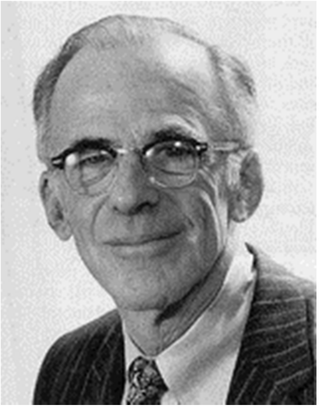
Science 60, 392 (1924)

Sputtered Tungsten “globules” from cathode entered the plasma

Hannes Olof Gösta Alfvén

On the Origin of the Solar System (1954)

The ionized gas condensed as small grains (or droplets) which moved in Kepler ellipses, and the planets were formed by the agglomeration of these grains.



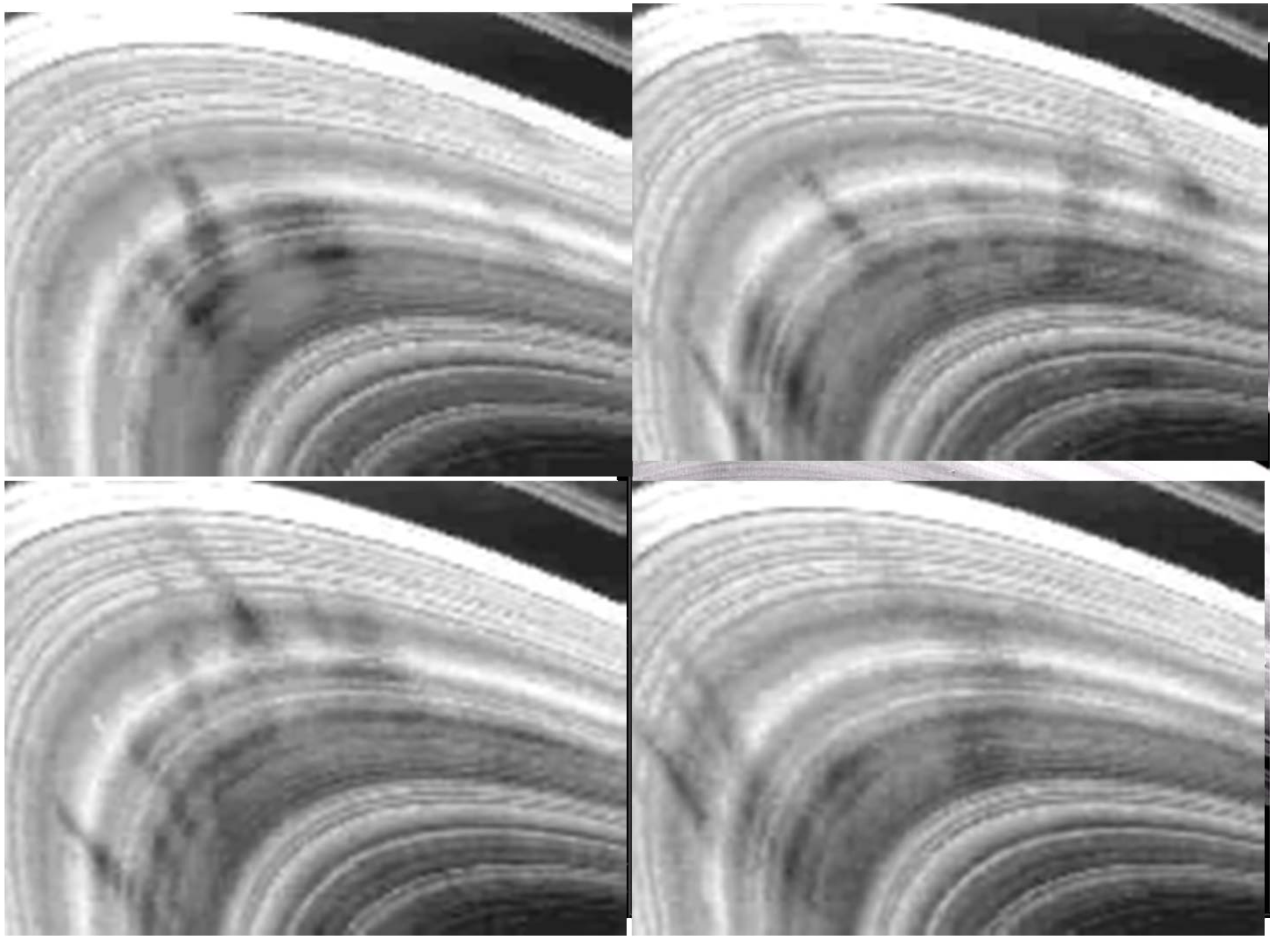
Lyman Spitzer, Jr.

Diffuse Matter in Space (1968).

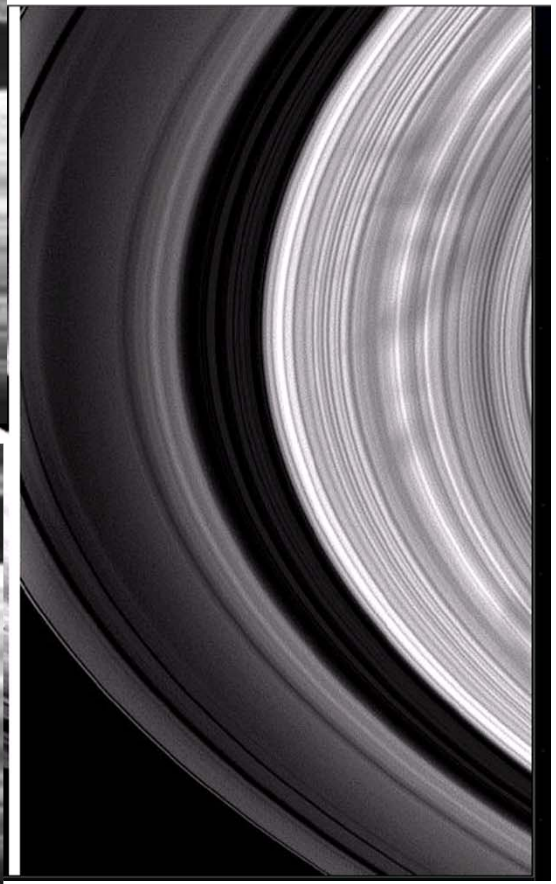
The photo-electric effect tends to make the grains positive, but the capture of swift electrons which hit the grains tend to give them a negative charge.

Saturn　Ring

Voyager Spacecraft 1981

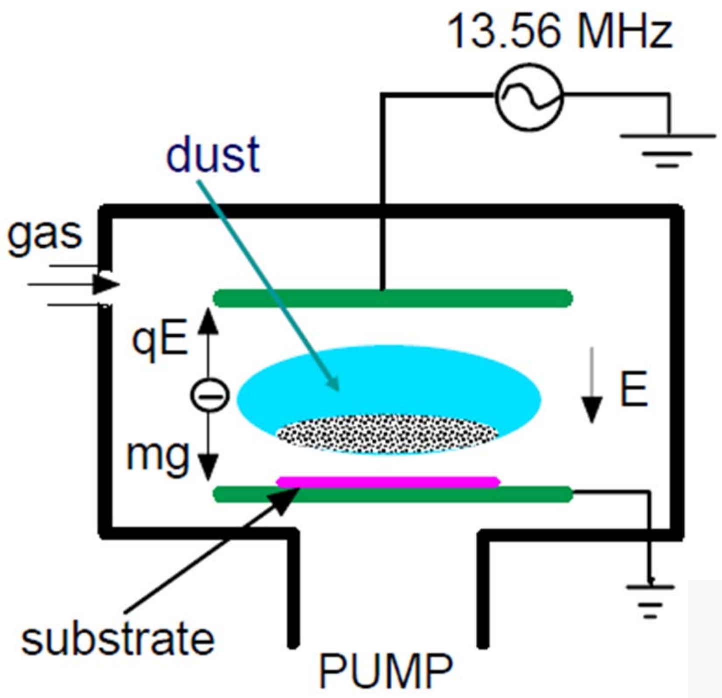


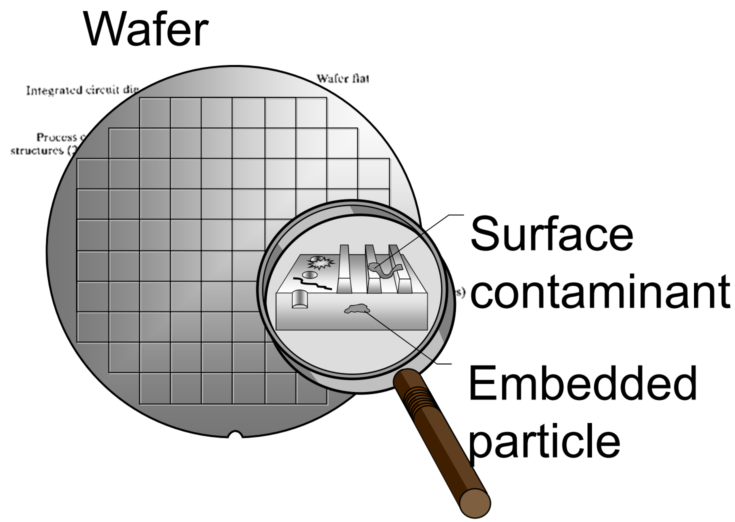
Cassini spacecraft
Nov. 2, 2008



Contamination Control

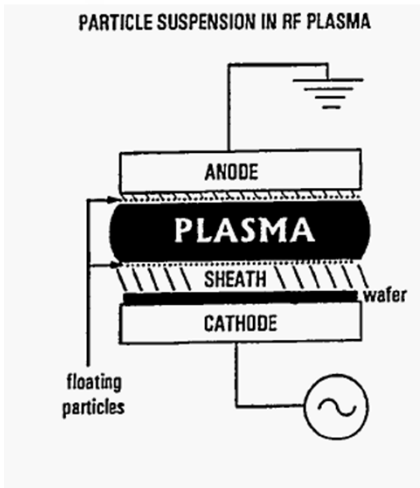
Plasma Processing in Microelecronics





silane (SiH_4) + Ar + $O_2 \rightarrow$ SiO_2 particles

IN SITU OBSERVATION



Particle suspension in a plasma

The 90% of the particles that end up on a wafer come about from the plasma (Selwyn, 1990)

Complex Plasmaの特徴

（１）比電荷

$$\frac{Q}{m_d}=\frac{\text{電荷量}}{\text{質量}}\approx 10^{-12}\left(\frac{e}{m_e}\right),\quad [1\mu\text{m},\quad -10^4 e]$$

（２） Q　変化量　　$Q=Q(\mathbf{x},t)$

（３）系全体として電気的中性

$$n_i=n_e+|Z_d|n_d$$

（４）結合系数

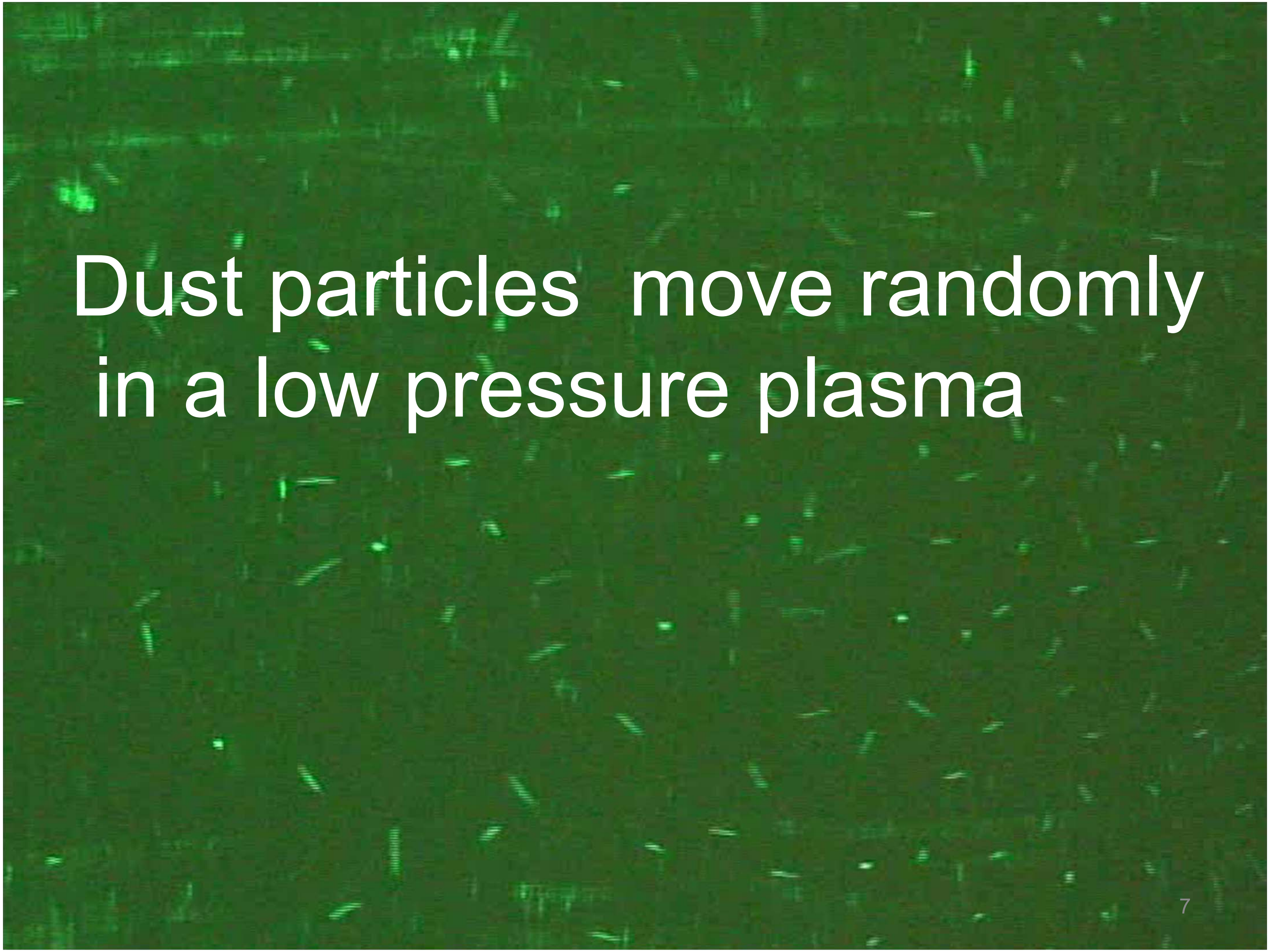
$$\Gamma=\frac{Z_d{}^2e^2/4\pi\varepsilon_0\Delta}{k_BT_d}e^{-\kappa}>>1$$

	質量、電荷
Electrons	m_e, $-e$
Ions	m_i, Z_ie ($Z_i\sim1$)
Neutrals	m_n, 0
Dust Particles	m_d, $Q=Z_de$

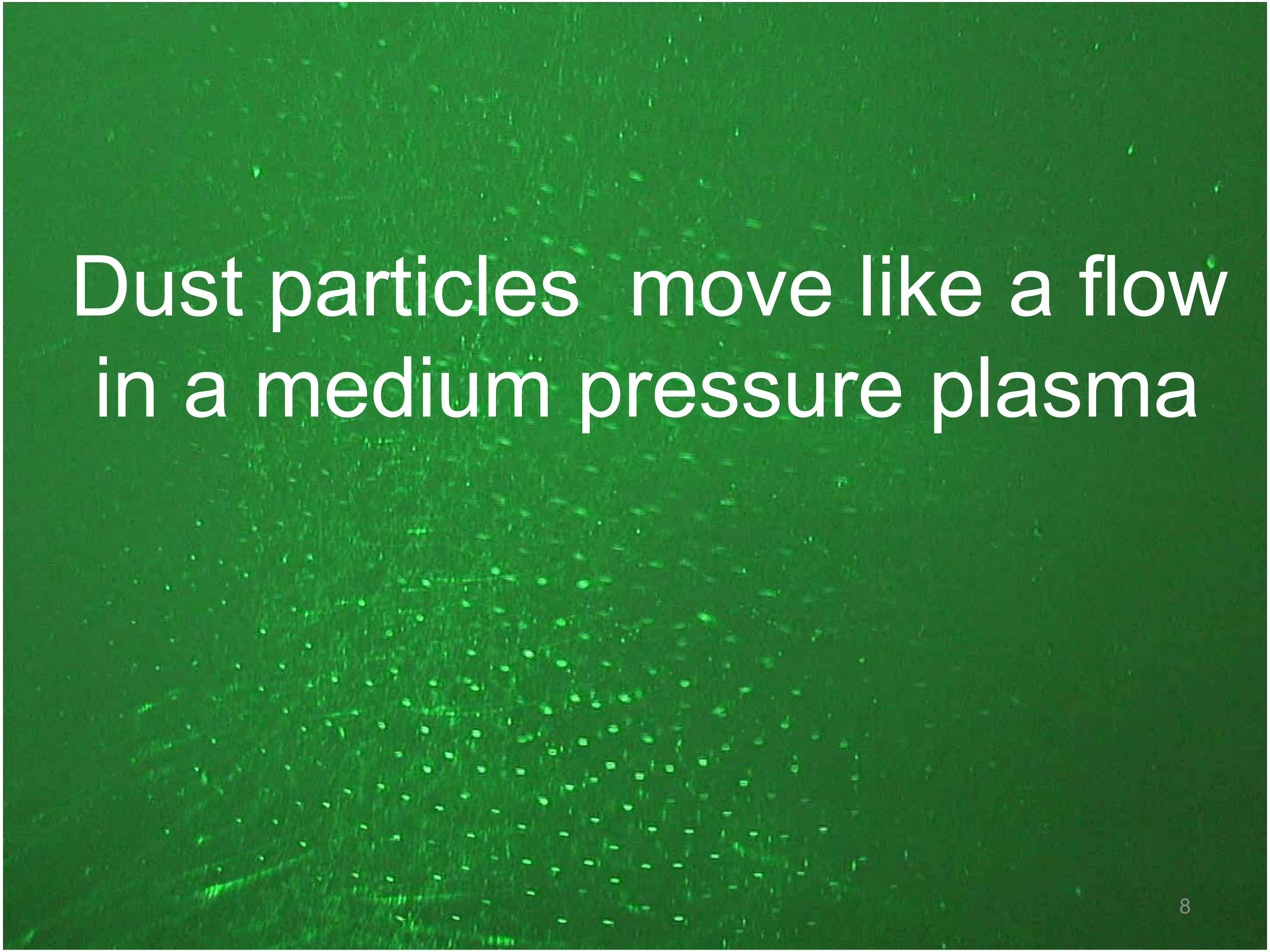
プラズマとダストの相互作用＝**Complex Plasma**

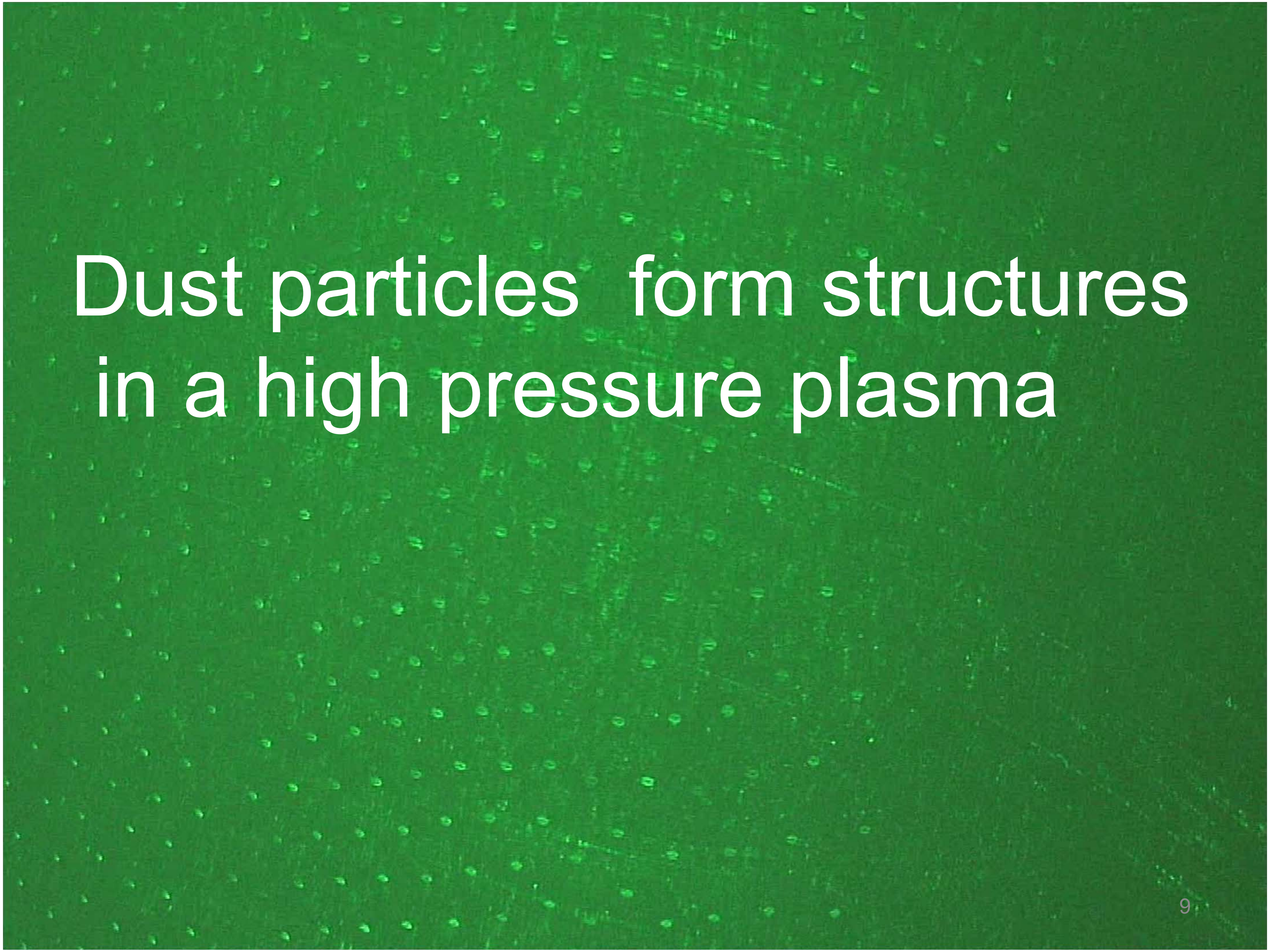
石原修，コンプレックスプラズマの物理、日本物理学会誌　2002年7月号 p. 476

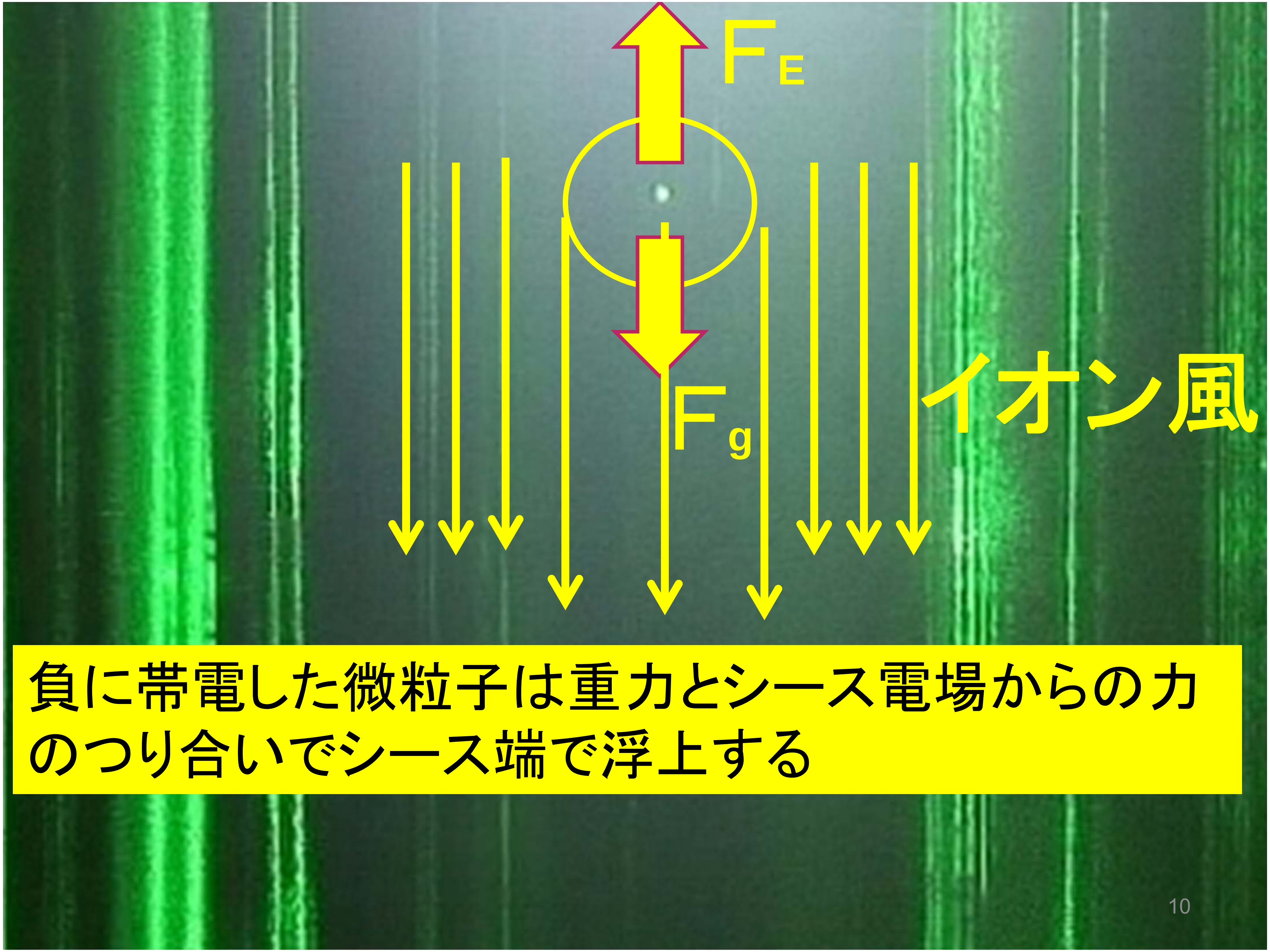
O. Ishihara, Complex Plasma : Dusts in Plasma, J. Phys. D: Appl. Phys. 40, R121 (2007).



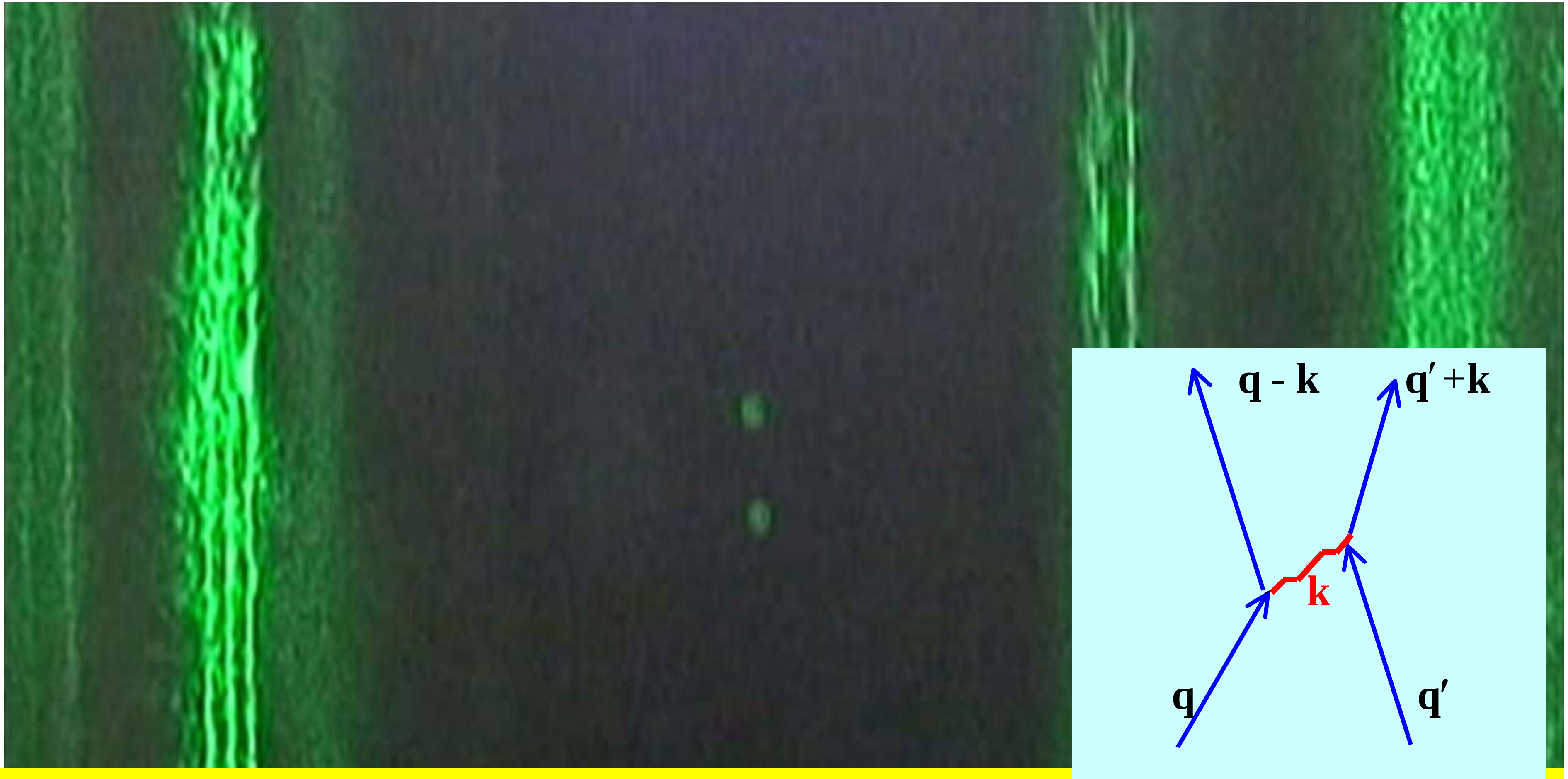
Dust particles move randomly in a low pressure plasma


Dust particles move like a flow
in a medium pressure plasma


Dust particles form structures
in a high pressure plasma



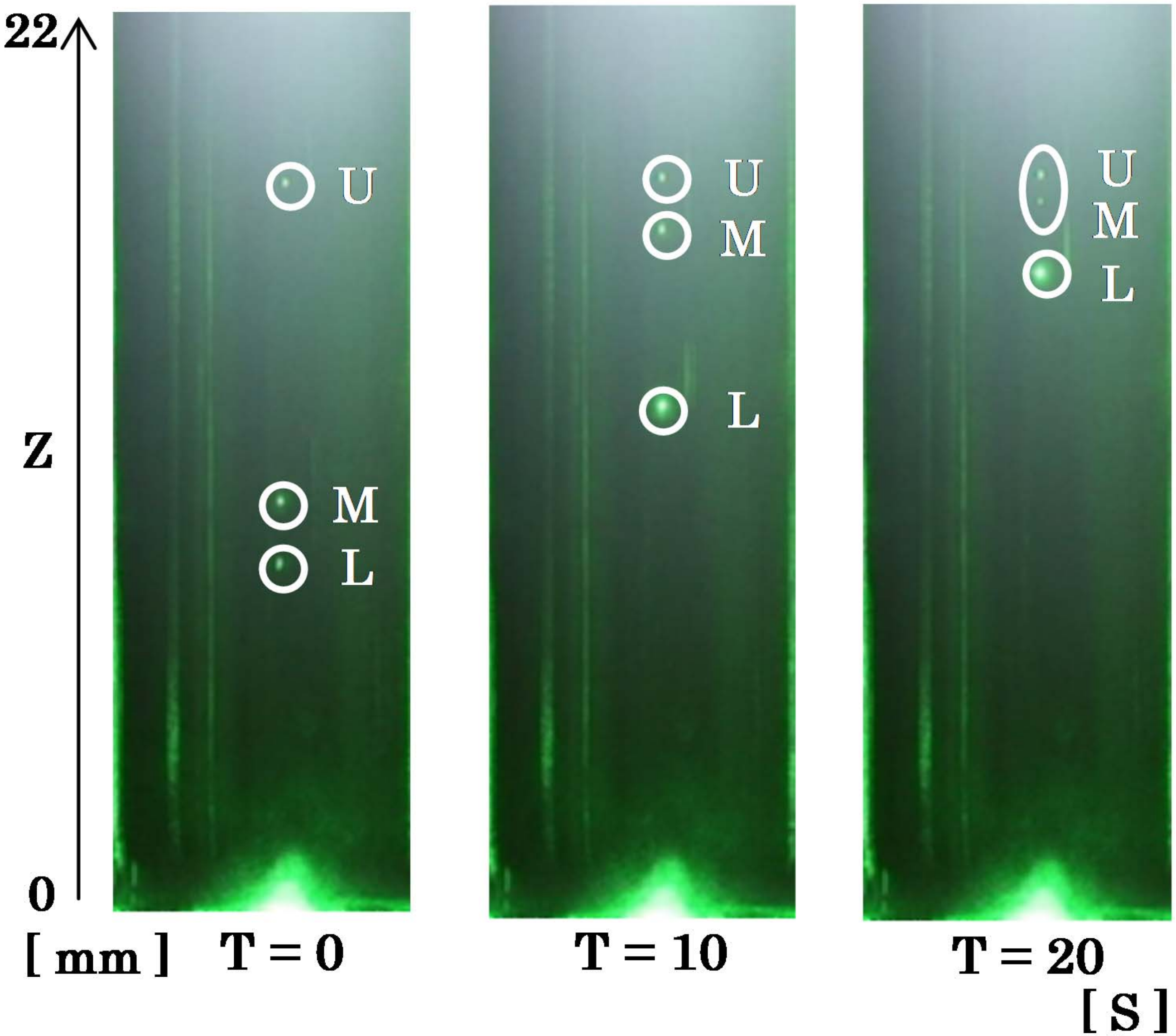
負に帯電した微粒子は重力とシース電場からの力のつり合いでシース端で浮上する

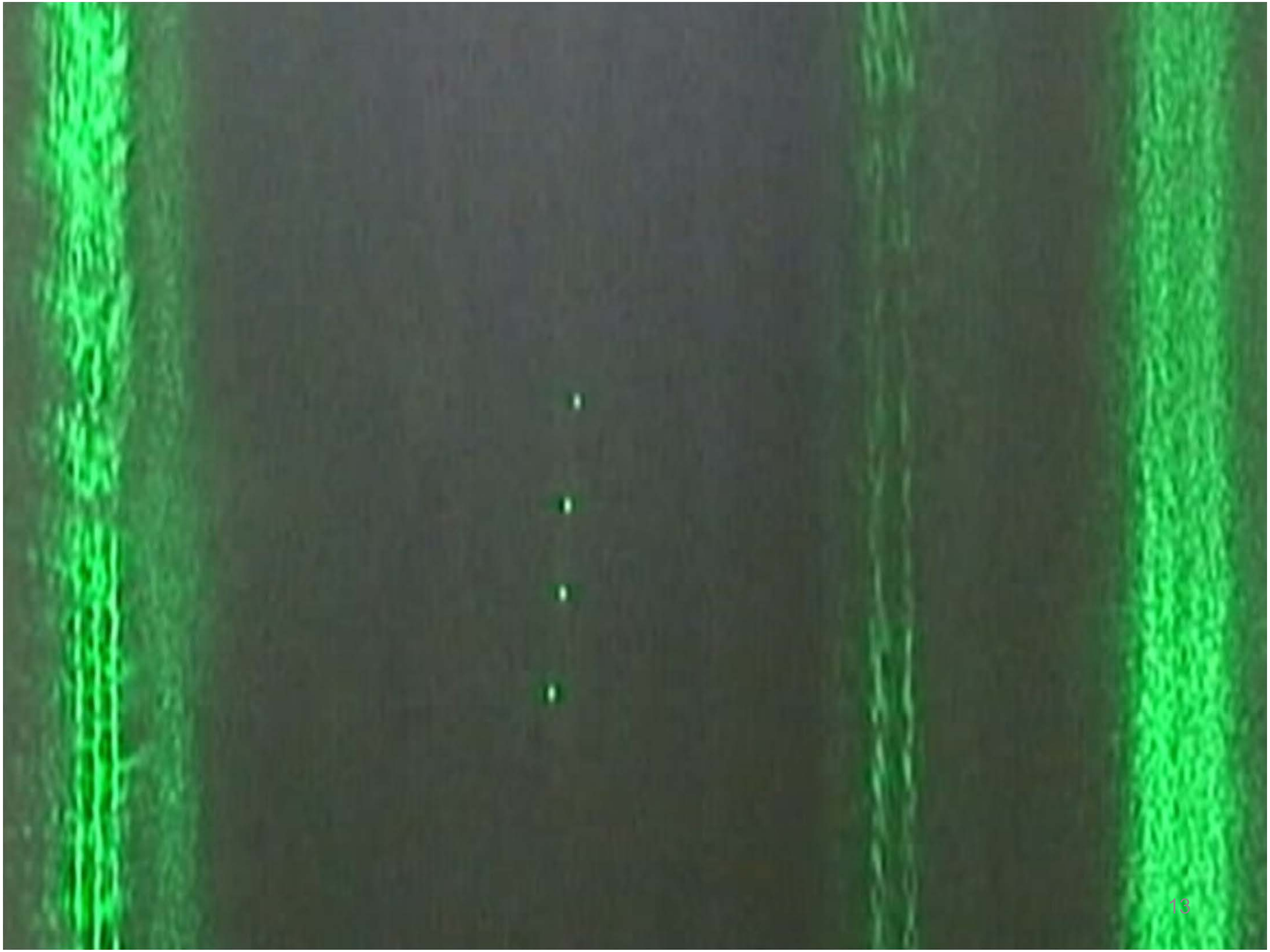


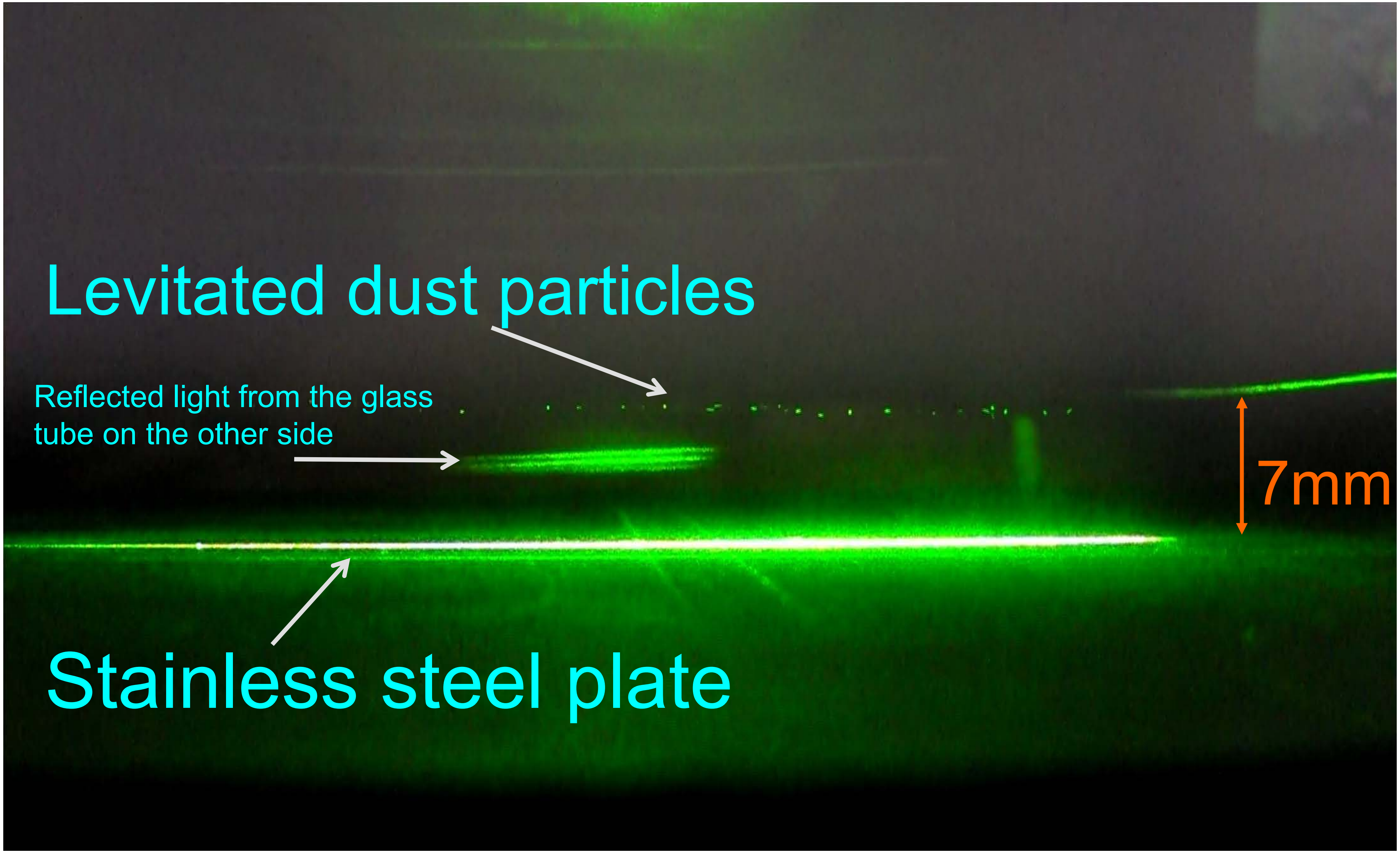
二つの微粒子がイオン風の中で
イオン音波を介してペアを作る

O. Ishihara and S.V. Vladimirov,
Physics of Plasmas (1996, 97, 98).

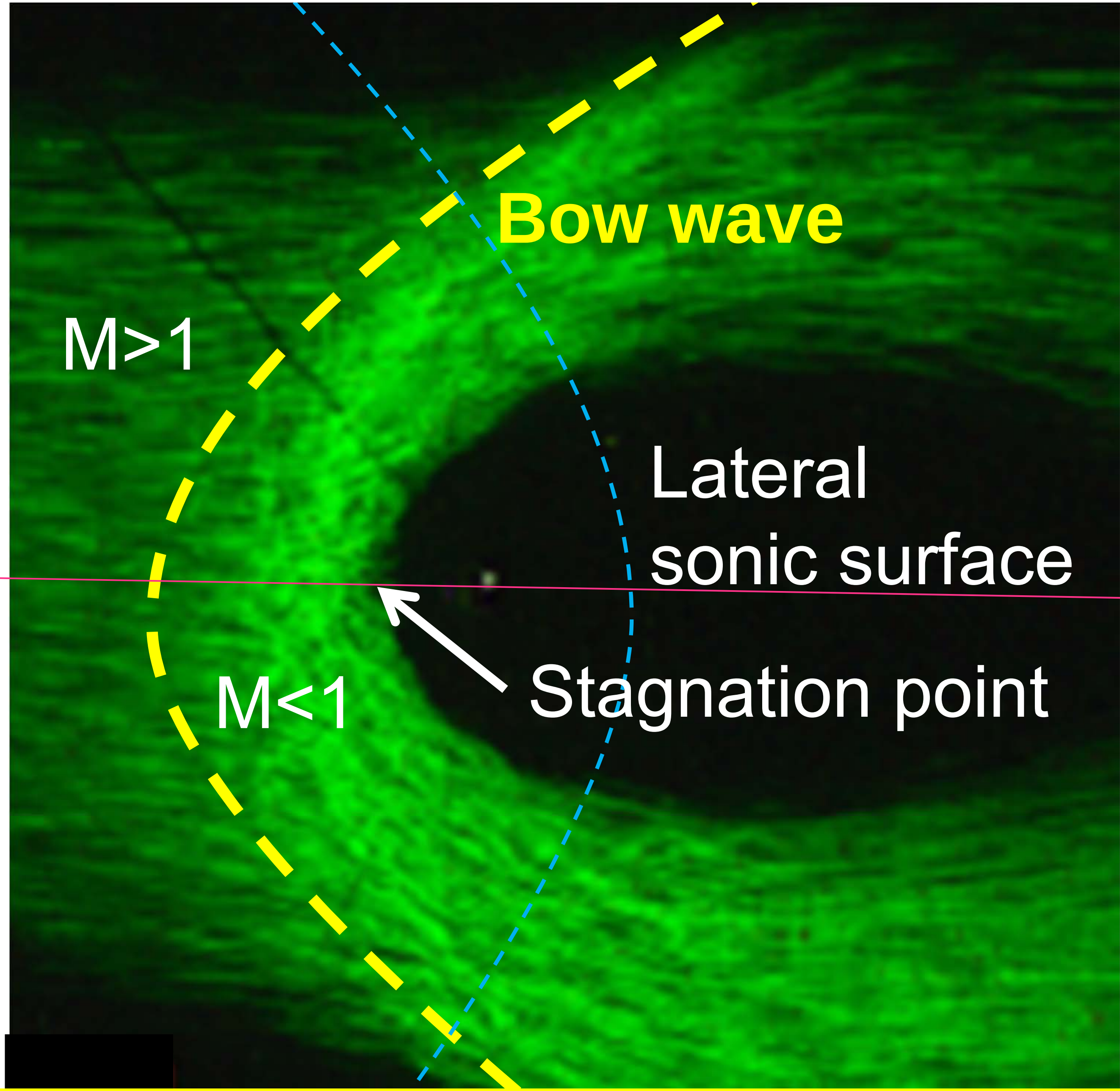
シース中の微粒子の挙動







Particles form a monolayer at the sheath edge above the stainless steel plate. YCOPEX （with Nakamura/Saitou）

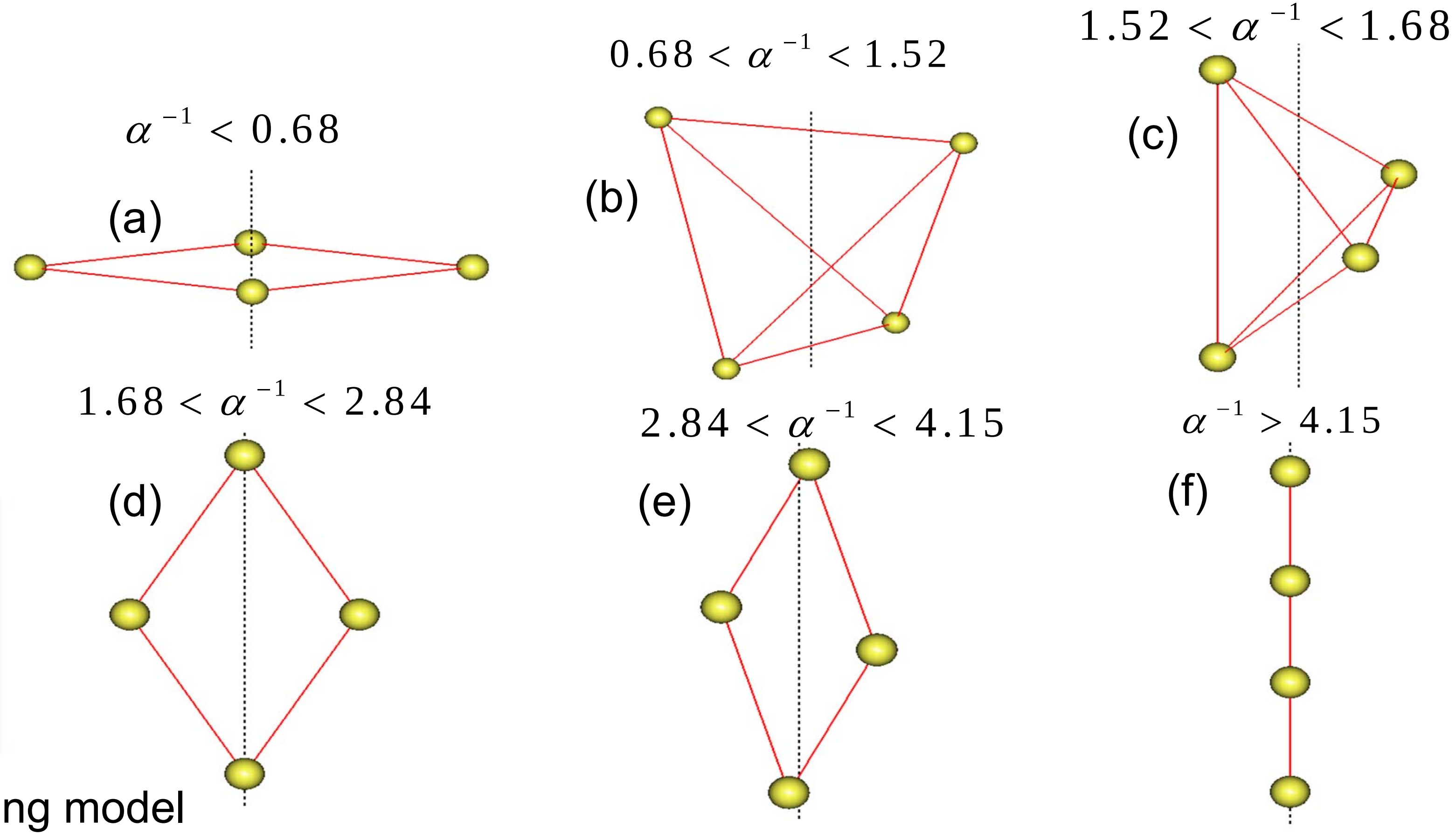


Y. Saitou, Y. Nakamura, T. Kamimura, and O. Ishihara,
Bow shock formation in a complex plasma, Phys. Rev. Lett. 108, 065004 (2012).

Molecular Dynamics Simulation

Minimum Energy Configuration N=4

$$H = \sum_{i=1}^{N}\left(x_i^2 + y_i^2 + \alpha z_i^2\right) + \frac{1}{2}\sum_{i,j=1}^{N}\frac{1}{\left|\mathbf{x}_i - \mathbf{x}_j\right|}$$

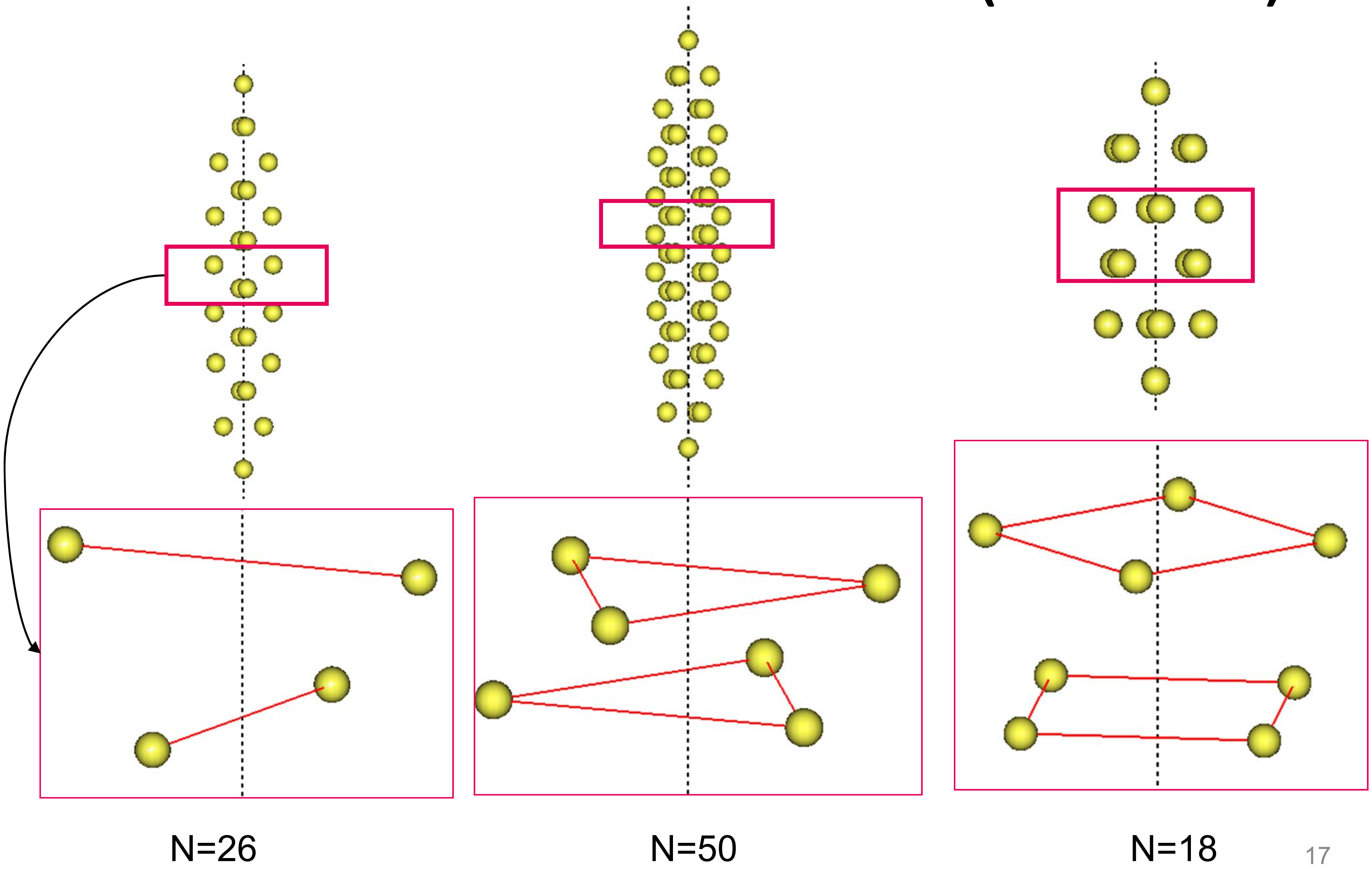


Plum pudding model

J.J. Thomson (1904) : atomic structure, Ishihara (1998) :

Kamimura & Ishihara (2007, 2012)

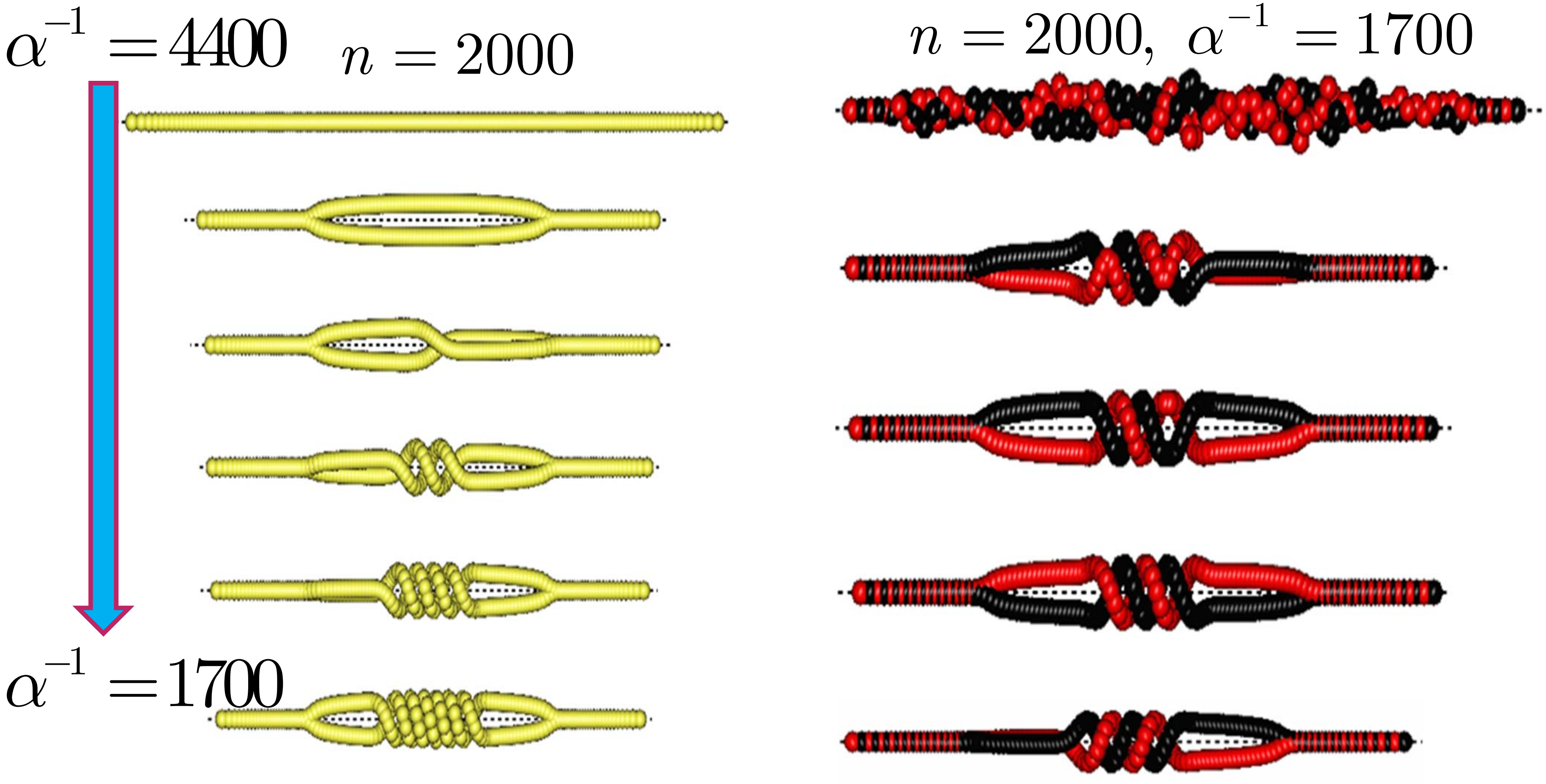
Fundamental structures ($\alpha^1 = 3.5$)



T Kamimura, Y Suga and O. Ishihara Phys. Plasmas 14, 123706 (2007)

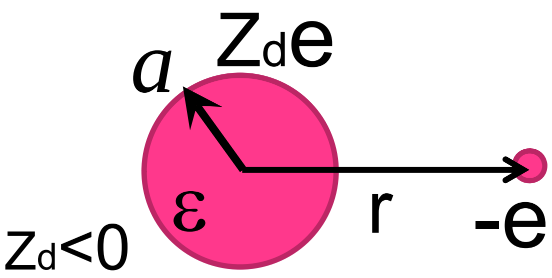
Molecular Dynamics Simulation

Double helical structure formed by microparticles in a complex plasma



T Kamimura and O Ishihara, Phys. Rev. E 85, 016406 (2012)
Truell W. Hyde et al., Collective Phenomena in Extended Vertical Chains within Dusty Plasma, ICPP, Stockholm, July 2012. O5.316

Interaction Energy (dust-electron)



$a \sim 1 \mu\text{m}$

$|Z_d| = 10^3 \sim 10^4$

$$\left|U_d\right|_{r\to a} = \left(\frac{|Z_d| e^2}{4\pi\varepsilon_0 r}\right)_{r\to a} = \frac{2|Z_d| a_B}{a} E_R$$

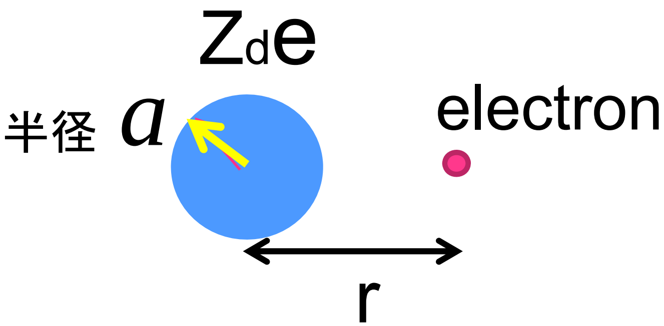
$$a_B = \frac{4\pi\varepsilon_0 \hbar^2}{m_e e^2} = 0.0529 \text{ nm} = \text{Bohr radius}$$

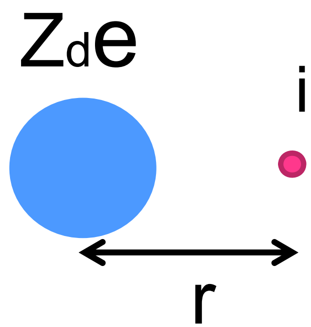
$$E_R = \frac{e^2}{8\pi\varepsilon_0 a_B} = 13.6 \text{ eV=Rydberg Energy}$$

For a=1μm and $|Z_d| = 1000$, $\left|U_d\right|_{r\to a} = 1.4$ eV.

Interaction involving dust particles

O. Ishihara, Inter. Cong. Plasma Physics, Stockholm, 2012 (Invited talk)





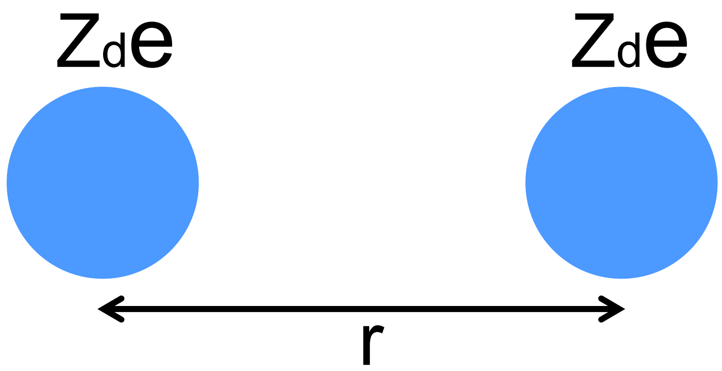
$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T}{ne^2}}$$

$$U_{\text{d-q}} = \frac{(Z_d e) q}{4\pi\varepsilon_0 r} \exp\left(-\frac{r}{\lambda_D}\right)$$

$$\tilde{F}_{\text{e(i)}} = -\frac{\partial U_{\text{e(i)}} / \partial r}{|Z_{\text{d}}| e^2 / 4\pi\varepsilon_0 a^2}$$

For r<<λ_{D},

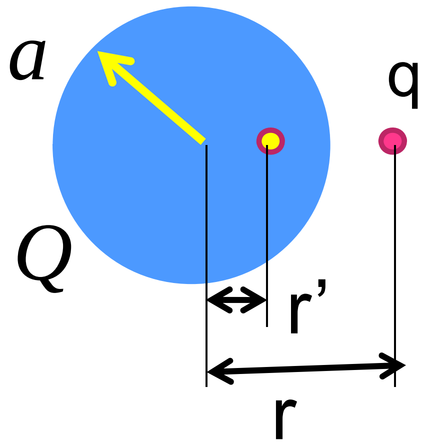
$$\tilde{F}_{\text{e}} = \left(\frac{a}{r}\right)^2, \qquad \tilde{F}_{\text{i}} = -\left(\frac{a}{r}\right)^2$$



$$U_{\text{d-d}} = \frac{(Z_d e)^2}{4\pi\varepsilon_0 r} \exp\left(-\frac{r}{\lambda_D}\right)$$

Lord Kelvin Problem (1848)

Charged Conducting Sphere and a Nearby Charge



$$q' = -\frac{a}{r}q,\ r' = \frac{a^2}{r}$$

$$F = \frac{q}{4\pi\varepsilon_0 r^2}\left(Q - qf(a,r)\right)$$

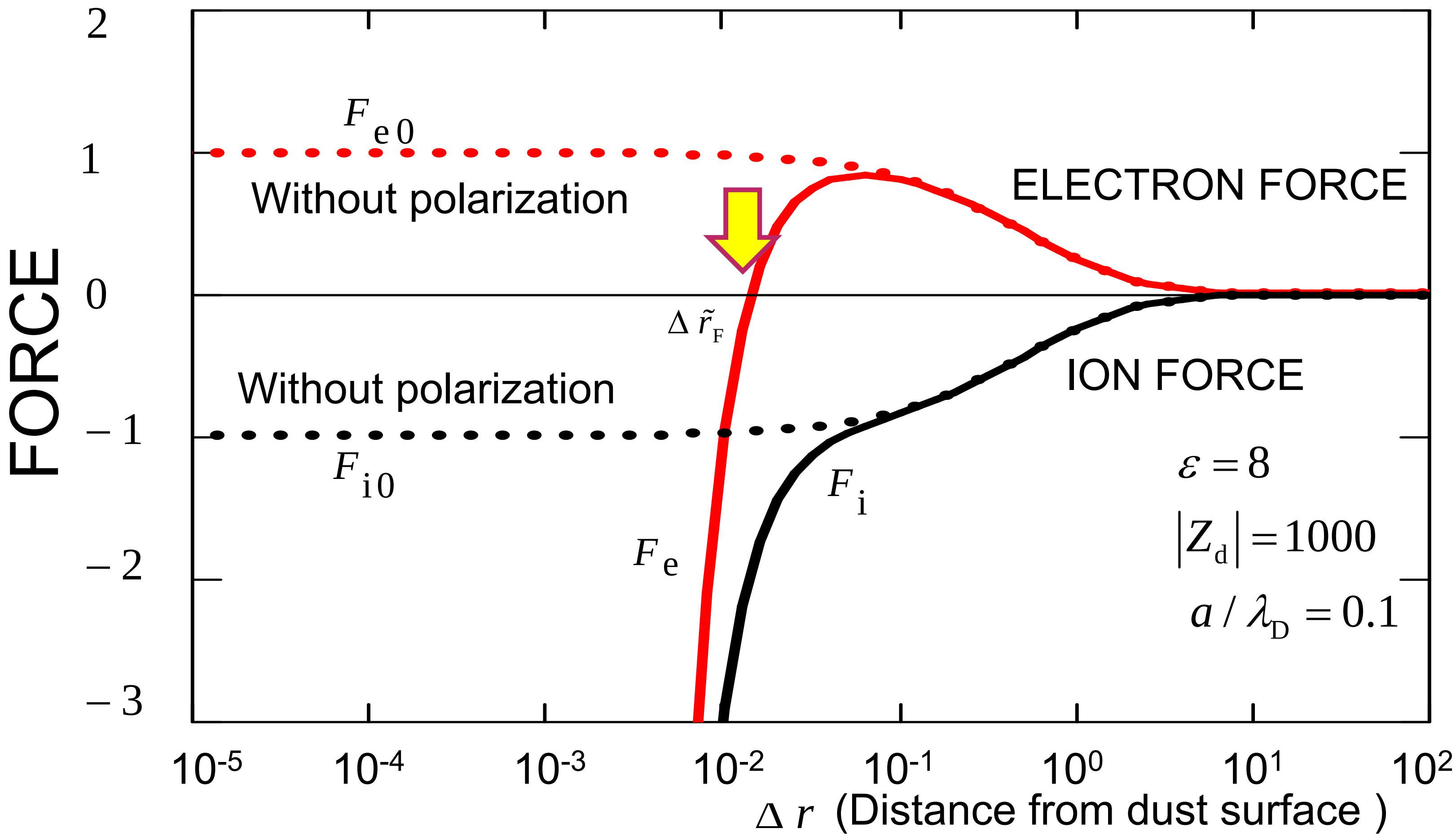
鏡像効果

$$F = 0 \text{ when } r \sim a\left(1 + \frac{1}{2}\sqrt{\frac{q}{Q}}\right)$$

qとQが同符号で、十分に近いところで反発力から引力に変わる

Electrons are attracted to a dust particle

when $\Delta r < \Delta r_{\mathrm{F}} = a / 2\sqrt{|Z_{\mathrm{d}}|}$

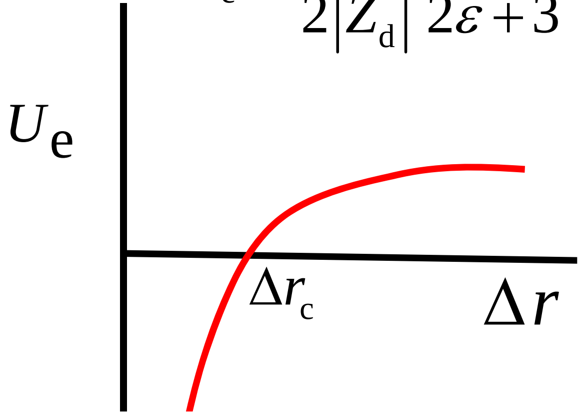


$$\tilde{F}_{\mathrm{e(i)}} = -\frac{\partial U_{\mathrm{e(i)}} / \partial r}{|Z_{\mathrm{d}}| e^2 / 4\pi\varepsilon_0 a^2} = \tilde{F}_{\mathrm{e(i)0}} + \Delta\tilde{F}_{\mathrm{e(i)}}$$

$$\Delta\tilde{r} = (r - a) / a << 1$$

Charge quantum effect (1)

$$\Delta r = r - a,\ \Delta r_c = \frac{a}{2|Z_d|}\frac{\varepsilon - 1}{2\varepsilon + 3}\exp(a/\lambda_D) \qquad U_e \approx U_d\left(1 - \frac{\Delta r_c}{\Delta r}\right),\ U_d = \frac{|Z_d|e^2}{4\pi\varepsilon_0 a}$$



DESORPTION TIME (Frenkel-Arrhenius parameterization)

$$\boxed{\tau_d = \frac{2\pi\hbar}{\kappa_t k_B T_d}\exp(E_d / k_B T_d)},$$

$$\frac{d\sigma_e}{dt} = \kappa_t I_e / S - \sigma_e / \tau_d$$

$$\frac{d\sigma_e}{dt} = 0 \Rightarrow \sigma_e = \kappa_t \tau_d I_e / S$$

$$Q = S\sigma_e,\ Q = \kappa_t \tau_d I_e$$

σ_e = surface charge density

κ_t = Transmission coefficient

T_d = Dust temperature

E_d = Desorption energy

$\tau_d = 1.8\mu s (T_d = 300K),$

$0.8s \quad (T_d = 170K)$ for $E_d = 0.42eV$

Ref. H. J. Kreuser and Z. W. Gortel, Physisorption Kinetics (1986).

Charge : quantum effect (2)

Schrödinger equation

EIGENENERGY: $E - U_\mathrm{d} = -\dfrac{1}{4n^2}\left(\dfrac{\varepsilon - 1}{2\varepsilon + 3}\right)^2 E_\mathrm{R} \exp(2a/\lambda_\mathrm{D})$

DESOPTION ENERGY($n=1$): $E_\mathrm{d} = \dfrac{1}{4}\left(\dfrac{\varepsilon - 1}{2\varepsilon + 3}\right)^2 E_\mathrm{R} \exp(2a/\lambda_\mathrm{D})$.

$$Q = Z_\mathrm{d} e = \kappa_\mathrm{t} \tau_\mathrm{d} I_\mathrm{e}$$

$$I_\mathrm{e} = -\frac{1}{4} e n_\mathrm{e} \bar{v}_\mathrm{e} S \exp(-z)$$

$$z = \frac{|Z_\mathrm{d}| e^2}{4\pi\varepsilon_0 a k_\mathrm{B} T_\mathrm{e}}$$

$$E_\mathrm{d} = \frac{1}{4}\left(\frac{\varepsilon - 1}{2\varepsilon + 3}\right)^2 E_\mathrm{R} \exp(2a/\lambda_\mathrm{D}).$$

$$Z_\mathrm{d} = -\frac{\pi\hbar}{2k_\mathrm{B} T_\mathrm{d}} n_\mathrm{e} \bar{v}_\mathrm{e} S \exp(-z + E_\mathrm{d}/k_\mathrm{B} T_\mathrm{d})$$

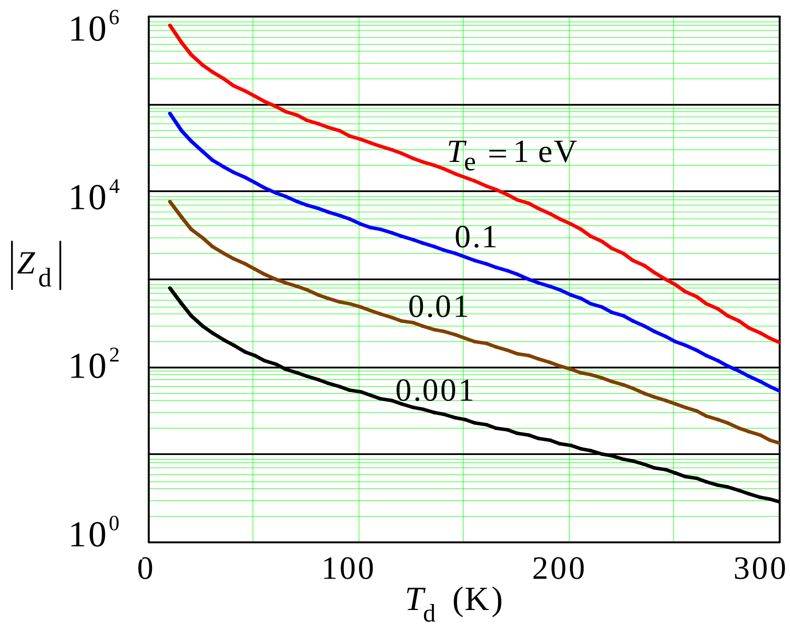
$$z = \sqrt{2\pi}\,\frac{a\lambda_\mathrm{dB}}{\lambda_\mathrm{D}^2}\,\frac{T_\mathrm{e}}{T_\mathrm{d}} \exp(-z + E_\mathrm{d}/k_\mathrm{B} T_\mathrm{d}).$$

Classical Theory (OML)

$$\exp(-z) = \sqrt{\frac{m_e T_i}{m_i T_e}}\left(1 + \frac{T_e}{T_i} z\right)$$

QM Theory (desorption)

$$\exp(-z) = \frac{z}{\sqrt{2\pi}} \frac{T_{\mathrm{d}}}{T_e} \frac{\lambda_{\mathrm{D}}^2}{a\lambda_{\mathrm{dB}}} \exp(-E_{\mathrm{d}} / k_{\mathrm{B}} T_{\mathrm{d}})$$



$$E_{\mathrm{d}} = \frac{1}{4}\left(\frac{\varepsilon - 1}{2\varepsilon + 3}\right)^2 E_{\mathrm{R}} \exp(2a / \lambda_{\mathrm{D}}).$$

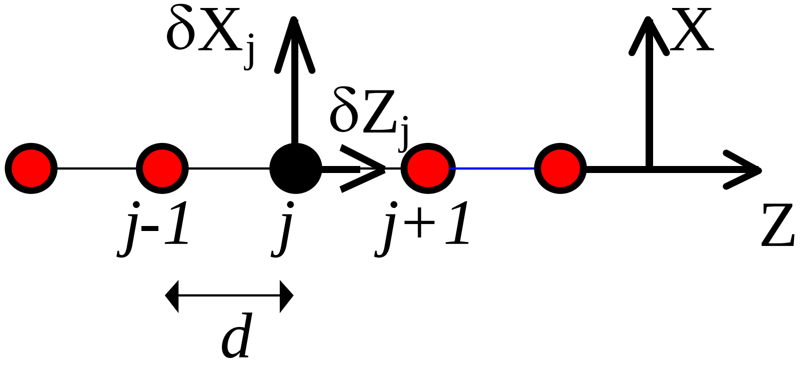
$$z = \frac{|Z_{\mathrm{d}}| e^2}{4\pi\varepsilon_0 a k_{\mathrm{B}} T_{\mathrm{e}}}$$

Dust charge as a function of dust temperature. Plasma density $n_{\mathrm{e}} = 10^{16}\ \mathrm{m}^{-3}$ and dust radius $a = 5\ \mu\mathrm{m}$. $E_{\mathrm{d}} = 0.2$ eV.

O. Ishihara, Plasma Physics and Controlled Fusion, in press (2012)

微粒子のプラズマ中での集団現象 ー格子振動

----- ダスト－プラズマ　クーロン相互作用



$$\phi(r) = \frac{Q}{4\pi\varepsilon_0 r} e^{-r/\lambda_D}$$

$$\mathrm{E}(\mathrm{r}) = -\nabla\phi(\mathbf{r}) = \mathrm{e}_r \frac{Q}{4\pi\varepsilon_0 r^2}\left(1+\frac{r}{\lambda_D}\right) e^{-r/\lambda_D}$$

$$\mathrm{E}(\mathrm{r}+\delta\mathrm{r}) \approx \mathrm{E}(\mathrm{r}) + \delta\mathrm{r}\cdot\frac{\partial}{\partial\mathrm{r}}\mathrm{E}(\mathrm{r})$$

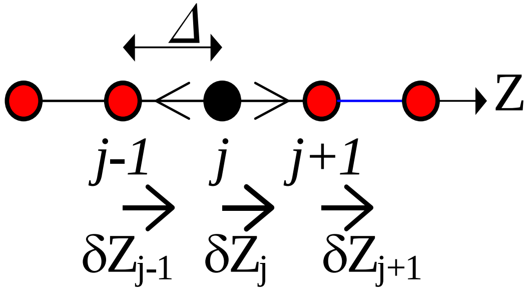
↓ 横振動

↓ 縦振動

O. Ishihara and S.V.Vladimirov,
Phys. Rev. E 57, 3392 (1998).

微粒子のプラズマ中での集団現象 ー格子振動

Longitudinal Mode



$$\delta Z_j = \delta Z_0 \exp\left[-i\left(\omega t - jk\Delta\right)\right]$$

$$\kappa = \Delta / \lambda_D$$

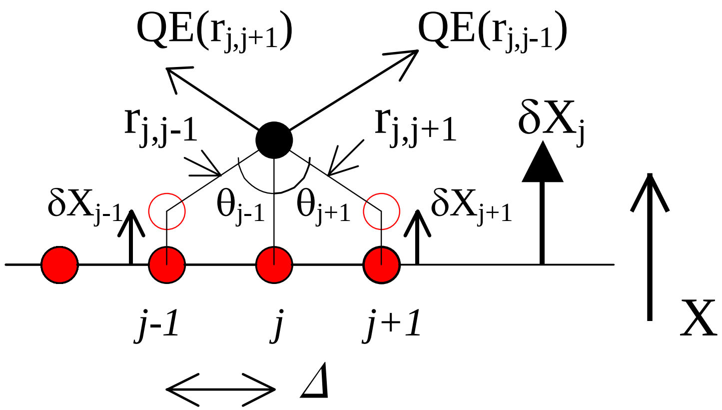
$$\left(\delta Z_j - \delta Z_{j-n}\right) + \left(\delta Z_j - \delta Z_{j+n}\right)$$
$$= \delta Z_0 e^{ink\Delta}\left[\left(1 - e^{-ink\Delta}\right) + \left(1 - e^{ink\Delta}\right)\right]$$
$$= \delta Z_0 e^{ink\Delta} 2\left[1 - \cos\left(nk\Delta\right)\right]$$
$$= \delta Z_0 e^{ink\Delta} 2\left[2\sin^2\left(\frac{nk\Delta}{2}\right)\right]$$
$$= 4\delta Z_0 e^{ink\Delta} \sin^2\left(\frac{nk\Delta}{2}\right)$$

$$QE(\mathrm{r}_j)$$
$$= Q\sum_{n=1}^{\infty}\left[(\delta Z_j \text{-} \delta Z_{j-n})\left(\frac{\partial E}{\partial r}\right)_{n\Delta} - (\delta Z_{j+n} \text{-} \delta Z_j)\left(\frac{\partial E}{\partial r}\right)_{n\Delta}\right]$$

$$m_d \frac{d^2 \delta Z_j}{dt^2} = Q\sum_{n=1}^{\infty}\left(2\delta Z_j - \delta Z_{j-n} - \delta Z_{j+n}\right)\left(\frac{\partial E}{\partial r}\right)_{n\Delta}$$

$$\omega^2 = \frac{8Q^2}{4\pi\varepsilon_0 m_d \Delta^3}\sum_{n=1}^{\infty}\left(\frac{1}{n^3} + \frac{\kappa}{n^2} + \frac{\kappa^2}{2n}\right)\exp\left(-n\kappa\right)\sin^2\left(\frac{nk\Delta}{2}\right)$$

Transverse Mode 一不安定



$$E(r_{j,j-n}) \approx E(r_{j,j+n}) \approx E(n\Delta)$$

$$\delta X_j = \delta X_0 exp\left[-i\left(\omega t - jk\Delta\right)\right]$$

$$QE(r_j) = Q\sum_{n=1}^{\infty}\left[E(r_{j,j-n})\cos\theta_{j-n} - E(r_{j,j+n})\cos\theta_{j+n}\right] = Q\sum_{n=1}^{\infty}\left[E(r_{j,j-n})\frac{\delta X_j - \delta X_{j-n}}{n\Delta} - E(r_{j,j+n})\frac{\delta X_j - \delta X_{j+n}}{n\Delta}\right]$$

$$QE(r_j) = Q\sum_{n=1}^{\infty} E(\mathrm{n}\Delta)\frac{2\delta X_j - \delta X_{j-n} - \delta X_{j+n}}{n\Delta}$$

$$\omega^2 = -\frac{4Q^2}{4\pi\varepsilon_0 m_d \Delta^3}\sum_{n=1}^{\infty}\frac{1}{n^2}\left(\frac{1}{n}+\kappa\right)\exp\left(-n\kappa\right)\sin^2\left(\frac{nk\Delta}{2}\right)$$

Transverse Lattice Mode Confined in a Plasma
－effect of charge neutrality

$$m_d \frac{d^2 \delta x_j}{dt^2} = QE(\Delta)\left(\frac{2\delta x_j - \delta x_{j-1} - \delta x_{j+1}}{\Delta}\right) - m_d \omega_0^2 \delta x_j$$

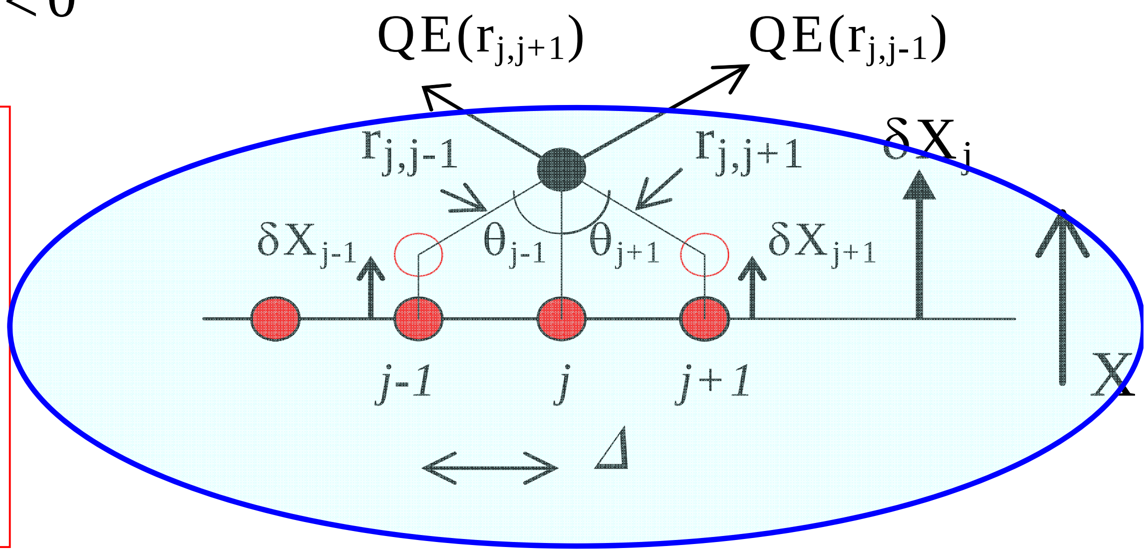
$$F = -m_d \omega_0^2 \delta x_j$$

$$\omega^2 = \omega_0^2 + a_T(\Delta)\sin^2\left(\frac{k\Delta}{2}\right),\ a_T(\Delta) < 0$$

$$\omega^2 < 0$$

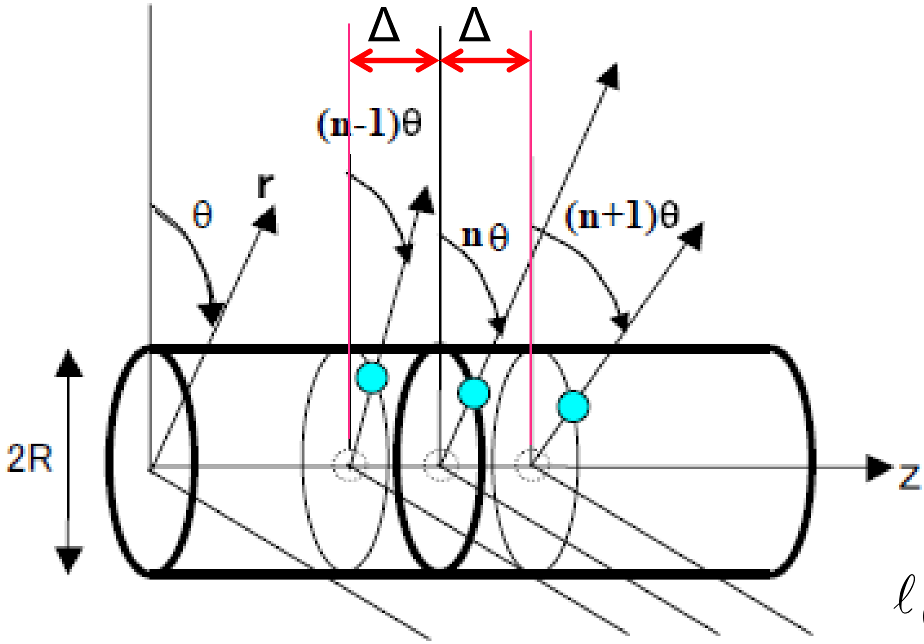
if $\omega_0^2 < -a_T(\Delta)$ or

$$\Delta < \Delta_{c0} \equiv 2^{\frac{2}{3}}\left(\frac{Q^2}{4\pi\varepsilon_0 m_d \omega_0^2}\right)^{1/3}$$



Unstable only if the interparticle distance becomes smaller than the critical value.

Linear chain to zig-zag, then to helical structure



$$\varepsilon = R / \Delta$$

$$\ell_0 \equiv (2Q^2 / 4\pi\varepsilon_0 m\omega_0^2)^{1/3}$$

$$H = \sum_{i=1}^{N} \left(x_i^2 + y_i^2 + \alpha z_i^2 \right) + \frac{1}{2} \sum_{i,j=1}^{N} \frac{1}{\left| \mathbf{x}_i - \mathbf{x}_j \right|}$$

$$E_0 \equiv m\omega_0^2 \ell_0^2 / 2$$

$$\mathbf{r}_n = (R\cos n\theta, R\sin n\theta, n\Delta)$$

$$\delta\rho_n, \delta\phi_n, \delta z_n \sim \exp\left[-i\left(\omega t + nk\Delta\right)\right]$$

Dispersion Relation

$$-\omega^2 \mathbf{1}' \cdot \delta\mathbf{R} = \frac{2}{\Delta^3} \sum_{j=1}^{N} \frac{\mathbf{C}_j \cdot \delta\mathbf{R}}{D_j^{3/2}}$$

$$\delta\mathbf{R} = \begin{pmatrix} \delta\rho \\ \varepsilon\delta\phi \\ \delta z \end{pmatrix} \qquad \mathbf{1}' = \begin{pmatrix} 1-\frac{2}{\omega^2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{C}_j = \text{Hermitian Matrix}$$

たとえば

$$\mathrm{C}_{11} = 1 - \cos(jk\Delta)\cos(j\theta) - \frac{24\varepsilon^2}{D_j}\cos^2\left(\frac{jk\Delta}{2}\right)\sin^4\left(\frac{j\theta}{2}\right)$$

Dispersion relation for a linear chain

$$\varepsilon = R/\Delta = 0$$

Longitudinal case

$$\omega^2 = \frac{8}{\Delta^3}\sum_{j=1}^{N}\frac{\sin^2\left(jk\Delta/2\right)}{j^3}$$

Transverse case

$$\omega^2 = 2 - \frac{4}{\Delta^3}\sum_{j=1}^{N}\frac{\sin^2\left(jk\Delta/2\right)}{j^3}$$

Linear chain is unstable for transverse modes if $\omega^2 < 0$

$$\Delta < \Delta_{c1}$$

$$\Delta_{c1} = \left[\sum_{j=1}^{N}\frac{2\sin^2\left(jk\Delta/2\right)}{j^3}\right]^{1/3} \xrightarrow[N\to\infty,\,k\Delta=\pi]{} \left(\frac{7\varsigma(3)}{4}\right)^{1/3} = 1.28131$$

$\varsigma(k) =$ Riemann zeta function

Linear chain transforms into double chains.

Zig-zag to Helices

Set $\delta\rho = \delta z = 0 \qquad \delta\phi \neq 0$

$$\omega^2 = -\frac{4}{\Delta^3}\sin^2\left(\frac{k\Delta}{2}\right) + \frac{2^4}{\Delta^3}\sin^2\left(\frac{k\Delta}{2}\right) - \frac{2}{\Delta^3}\sin^2\left(\frac{k\Delta}{2}\right)\cos^2\left(\frac{k\Delta}{2}\right) + \cdots$$

$$\omega^2 < 0 \text{ if } 3>8\left[1+\left(2D/\Delta\right)^2\right]^{-3/2}$$

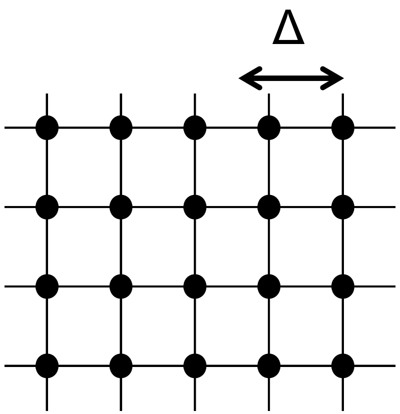
$$\text{Substitute } D = \left[-1+\frac{2^{8/3}}{\left(\Delta^3-16S\right)^{2/3}}\right]^{1/2}\frac{\Delta}{2}$$

$$\text{Then } \boxed{\Delta < \left[-10+14\varsigma(3)\right]^{1/3} = 1.897 = \Delta_{c2}}$$

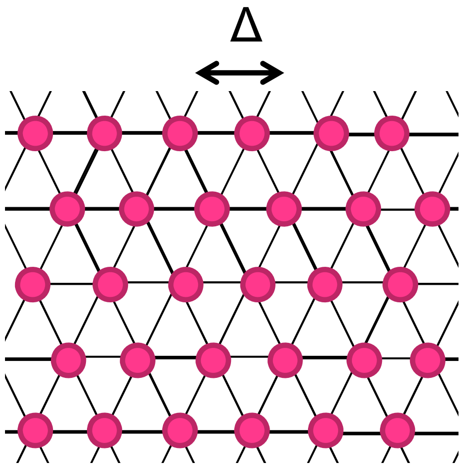
Zig-zag structure transforms into helical structur

T Kamimura and O Ishihara, Physical Review E 85, 016406 (2012)

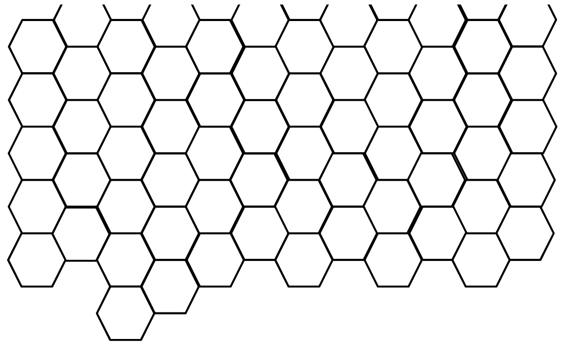
Two dimensional lattices



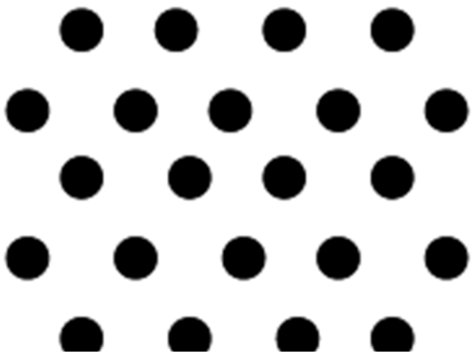
square lattice



hexagonal lattice



honeycomb lattice



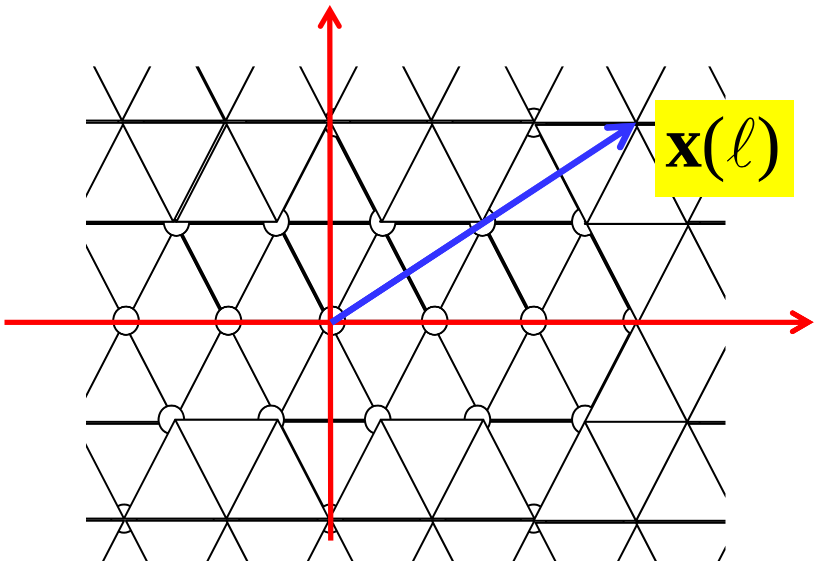
Interaction Energy

Consider interaction energy of a given dust particle at the origin with all other dust particles in the crystal in the presence of plasma.

$$E_I = \text{total interaction energy} = E_I^Q + E_I^b$$

E_I^Q = interaction energy between dust particles

E_I^b = interaction energy of the reference dust particle with the neutralizing background

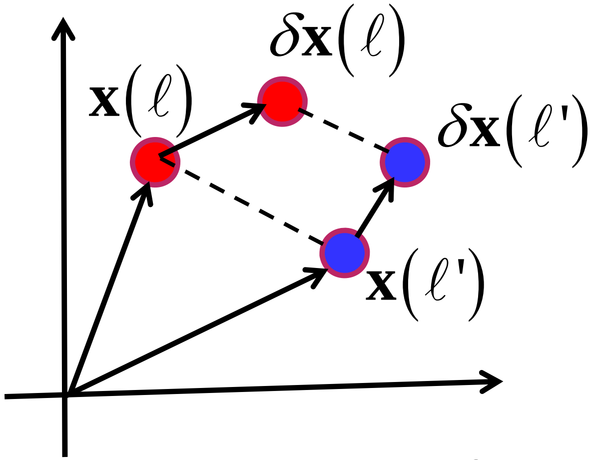


$$E_I^Q = \frac{Q^2}{4\pi\varepsilon_0} \lim_{\mathbf{x}\to 0} \sum_{\ell} \left(\frac{e^{-|\mathbf{x}-\mathbf{x}(\ell)|/\lambda}}{|\mathbf{x}-\mathbf{x}(\ell)|} - \frac{e^{-|\mathbf{x}|/\lambda}}{|\mathbf{x}|} \right)$$

$$E_I^b = Q\int \frac{\rho_{\mathrm{Q}}}{4\pi\varepsilon_0 |\mathbf{x}|} e^{-|\mathbf{x}|/\lambda} d^2x = -\frac{Q^2}{4\pi\varepsilon_0 a_c} 2\pi\lambda$$

Background charge density $\rho_{\mathrm{Q}} = -\dfrac{Q}{a_c}$

Variation of Potential Energy



$$\mathbf{x}_{\ell\ell'} = \mathbf{x}(\ell) - \mathbf{x}(\ell')$$

$$\delta\mathbf{x}_{\ell\ell'} = \delta\mathbf{x}(\ell) - \delta\mathbf{x}(\ell')$$

$$\Phi = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0}\sum_{\ell,\ell'} \frac{e^{-\left(\left[\mathbf{x}(\ell)+\delta\mathbf{x}(\ell)\right]-\left[\mathbf{x}(\ell')+\delta\mathbf{x}(\ell')\right]\right)/\lambda}}{\left[\mathbf{x}(\ell)+\delta\mathbf{x}(\ell)\right]-\left[\mathbf{x}(\ell')+\delta\mathbf{x}(\ell')\right]}$$

$$\Phi = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0}\sum_{\ell,\ell'} \frac{e^{-\left|\mathbf{x}_{\ell\ell'}+\delta\mathbf{x}_{\ell\ell'}\right|/\lambda}}{\left|\mathbf{x}_{\ell\ell'}+\delta\mathbf{x}_{\ell\ell'}\right|}$$

$$= \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0}\sum_{\ell,\ell'} \left[\frac{e^{-\left|\mathbf{x}_{\ell\ell'}\right|/\lambda}}{\left|\mathbf{x}_{\ell\ell'}\right|} + \delta\mathbf{x}_{\ell\ell'}\cdot\frac{d}{d\mathbf{x}_{\ell\ell'}}\left(\frac{e^{-\left|\mathbf{x}_{\ell\ell'}\right|/\lambda}}{\left|\mathbf{x}_{\ell\ell'}\right|}\right) + \frac{1}{2}\left(\delta\mathbf{x}_{\ell\ell'}\cdot\frac{d}{d\mathbf{x}_{\ell\ell'}}\right)^2\left(\frac{e^{-\left|\mathbf{x}_{\ell\ell'}\right|/\lambda}}{\left|\mathbf{x}_{\ell\ell'}\right|}\right) + \cdots\right]$$

Dispersion Relation for hexagonal lattice

For $\kappa << 1$

$$\frac{\omega}{k} = \sqrt{\frac{2\pi}{\sqrt{3}}}\sqrt{\frac{2Q^2}{4\pi\varepsilon_0 m_d \Delta}}\frac{1}{\sqrt{\kappa}} = 1.90 C_s \frac{1}{\sqrt{\kappa}}$$

For $\kappa >> 1$

$$\frac{\omega}{k} = \sqrt{\frac{4\sqrt{2}\pi^2}{3^{3/4}}}\sqrt{\frac{2Q^2}{4\pi\varepsilon_0 m_d \Delta}}\kappa^{-5/2} e^{-\sqrt{3}\kappa^2/8\pi}$$

$$= 4.95 C_s \kappa^{-5/2} e^{-\sqrt{3}\kappa^2/8\pi}$$

$$\kappa = \frac{\Delta}{\lambda_D}$$

$$C_s = \sqrt{\frac{2Q^2}{4\pi\varepsilon_0 m_d \Delta}}$$

O. Ishihara, Plasma Physics and Controlled Fusion (2012)

SUMMARY

１．コンプレックスプラズマの実験的側面

微粒子が作る気体、液体、固体状態

プラズマの集団運動を介した微粒子間相互作用

２．微粒子が形成する構造

バウショック

MEC（Minimum Energy Configuration）

３．微粒子の帯電過程

鏡像・分極効果, 吸着、脱離、量子効果

４．コンプレックスプラズマ中の低次元集団運動

Complex Plasma

an emerging field

END