Self-Organization of plasma Turbulence and Reduction of Anomalous Transport

日本物理学会第67年次大会

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講演概要　1

- 流体の振る舞いを記述する古典的な方程式にNavier-Stokes
- Equation がある。この方程式の特徴は0 周波数以外の固有振動を
- 持たず、粘性による散逸が小さい極限では全ての擾乱が非線形とな
- り乱流状態を作る点にある。特に興味ある事実は、流れが2 次元に
- 制約されている場合、その面に垂直方向の渦度が保存され、その結
- 果乱流のエネルギースペクトルが波数の小さい方向に逆カスケー
- ドし、凝縮することである。磁場プラズマ中の低周波の静電擾乱も
- 似た性質を持ち乱流スペクトルは逆カスケードする(A.Hasegawa
- and K. Mima, Phys. Fluids, Vol.21, 87 (1978)) 。磁場中プラ
- ズマは磁場に垂直な面で不均一となる為、この不均一性に起因する
- 固有振動（ドリフト波）が存在するが、電磁波による散乱実験観測
- 結果ではプラズマは常時乱流状態にあり、周波数スペクトルの幅は
- ドリフト波の固有振動数を大きく上回る事が知られている。

講演概要　２

- 乱流スペクトルが波長の小さい方向に逆カスケードする模様はプ
- ラズマの持つ境界条件や他の制約条件によって異なる。円筒プラズ
- マの場合は円周方向の波長は０ 、半径方向の波長は角運動量の保存
- などの制約条件により、有限の値に凝縮する(A.Hasegawa et. al,
- Phys.Fluids. vol.22, 2122 (1979), A.Hasegawa and M.
- Wakatani, Phys. Rev. Lett. vol.59, 1581 (1987))。この結果、
- 凝縮された乱流スペクトルは円周方向に流れる帯状流を発生する。
- プラズマの流れは等電位面に沿う為、帯状流の発生は円周方向に一
- 様な等電位が存在する事を意味する。このため、等電位面に沿って
- 動く電子の半径方向の動きは低減され、プラズマの閉じ込めが良く
- なる。従来プラズマ乱流は異常拡散を誘起すると考えられていたが、
- 長谷川らの予想はこの逆で、乱流が自己組織化することにより、プ
- ラズマの閉じ込が改良されるというものであった。１９７０年から
- ８０年代に渡って築き上げられた乱流によるプラズマ閉じ込め理
- 論は２１世紀に入り多くの実験やシミュレーションで確認される
- に至ったのである。

Theoretical Consideration: What is the origin for large nonlinearity to cause the large spectral width and the density fluctuations?

The density fluctuation requires compressibility,

$$\nabla \bullet v \neq 0$$

which is served by the polarization drift of ions.

$$\mathbf{v}_p = \frac{1}{\omega_{ci} B_0} \frac{d\mathbf{E}_\perp}{dt} = \frac{1}{\omega_{ci} B_0} \left(\frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla \right) \mathbf{E}_\perp$$

Here $\mathbf{v}_E$ is the **ExB** drift velocity. The nonlinear polarization term is equivalent to the convection of vorticity, so we use equation of ion vorticity;

$$\Omega = (\nabla \times \mathbf{v}_\perp) \cdot \hat{\mathbf{z}} = \nabla \times \left(\frac{-\nabla \phi \times \hat{\mathbf{z}}}{B_0} \right) = \frac{\nabla^2 \phi}{B_0}$$

The equation for ion vorticity

- Take the curl of ion equation of motion:

$$\frac{d}{dt}\left(\Omega+\omega_{ci}\right)+\left(\Omega+\omega_{ci}\right)\nabla\cdot\mathbf{v}_\perp=0$$

while the equation of continuity gives;

$$\nabla\cdot\mathbf{v}_\perp=-\frac{d}{dt}\ln n=-\frac{d}{dt}\left(\ln n_0+\frac{e\phi}{T_e}\right)$$

The equation for potential vorticity now reads;

$$\frac{d}{dt}\left(\ln\frac{\omega_{ci}}{n_0}+\frac{\Omega}{\omega_{ci}}-\frac{e\phi}{T_e}\right)\equiv\left(\frac{\partial}{\partial t}-\frac{\nabla\phi\times\widehat{\mathbf{z}}}{B_0}\cdot\nabla\right)\left(\ln\frac{\omega_{ci}}{n_0}+\frac{\Omega}{\omega_{ci}}-\frac{e\phi}{T_e}\right)=0.$$

The Model Equation (The Hasegawa-Mima Equation)

Normalization:

$$\varepsilon\omega_{ci}t \equiv t,\ x/\rho_s \equiv x,\ e\phi/T_e \equiv \varepsilon\phi,\ where\ \rho_s = \sqrt{T_e/m_i}\,/\,\omega_{ci}$$

The Medel Equation (Hasegawa-Mima Equation) is derived in $O(\varepsilon^2)$:

$$\frac{\partial}{\partial t}\left(\nabla^2\phi - \phi\right) - \left[\left(\nabla\phi \times \hat{\mathbf{z}}\right)\cdot\nabla\right]\left[\nabla^2\phi - \frac{1}{\varepsilon}\ln\left(\frac{n_0}{\omega_{ci}}\right)\right] = 0.$$

Conservation Laws and Inverse Cascade

Total Energy:

$$\frac{\partial W}{\partial t} \equiv \frac{\partial}{\partial t} \int \left[\left(\nabla \phi \right)^2 + \phi^2 \right] dV = 0$$

Total (potential) Enstrophy:

$$\frac{\partial U}{\partial t} \equiv \frac{\partial}{\partial t} \int \left[\left(\nabla \phi \right)^2 + \left(\nabla^2 \phi \right)^2 \right] dV = 0$$

Because of existence of the two global conserved quantities, in the presence of dissipation, the enstrophy selectively decays and energy spectrum cascades to smaller wavenumbers (Inverse cascades, Kraichnan, Phys. Fluids, vol. 10, 1417). In a toroidal plasma, the inverse cascade will result in the formation of pololoidal zonal flows (A Condensed State).

Conservation Laws and Inverse Cascade

Boltzmann Statistics:

$$\delta\left(\int f \ln f \, dV - \lambda_1 W - \lambda_2 U\right) = 0$$

giving,

$$f \sim \exp\left(-\lambda_1 W - \lambda_2 U\right)$$

and

$$\left\langle W_k \right\rangle = \frac{1}{\lambda_1 + k^2 \lambda_2} : Possibility\ of\ a\ negative\ temperature$$

Conservation Laws and Inverse Cascade

Self-organized state in the presence of dissipation:

$$\delta\left(\int U - \lambda W\right) = 0$$

giving,

$$\nabla^2\phi + \lambda\phi = 0: \quad \textit{Eigenvalue equation for } \phi$$

Inverse Cascades
-Weak Turbulence Regime

In Weak Turbulence Regime:

$$If\ \omega_{nl} << \omega_{\mathbf{k}}, the\ wave\ quanta\ N_{\mathbf{k}}\ is\ conserved:$$

$$\omega_{\mathbf{k}} = \frac{(\mathbf{k}\times\hat{\mathbf{z}})\cdot\nabla\ln(n_0/\omega_{ci})}{\varepsilon(1+k^2)}$$

$$and\ N_p = (1+k_p^2)|\phi_p|^2/|k_q^2 - k_r^2|,\ \ k_p^2 \neq k_r^2$$

$$If\ we\ take;\ k_1^2 \le k_2^2 \le k_3^2, we\ have$$

$$N_3 - N_1 = const.,\ \ N_1 + N_2 = const.,\ \ N_2 + N_3 = const.$$

➔Dual Cascade of Energy.

Inverse Cascades
-Weak Turbulence Regime

In a region of small wave numbers, the resonant three wave interactions take place since

$$k_q^2 - k_r^2 = k_{qy} / \omega_q - k_{ry} / \omega_r = \omega_p M$$

$$where\ k_y\ is\ the\ wavenumber\ in\ the\ direction\ of\ \hat{\mathbf{z}} \times \nabla \ln n_0$$

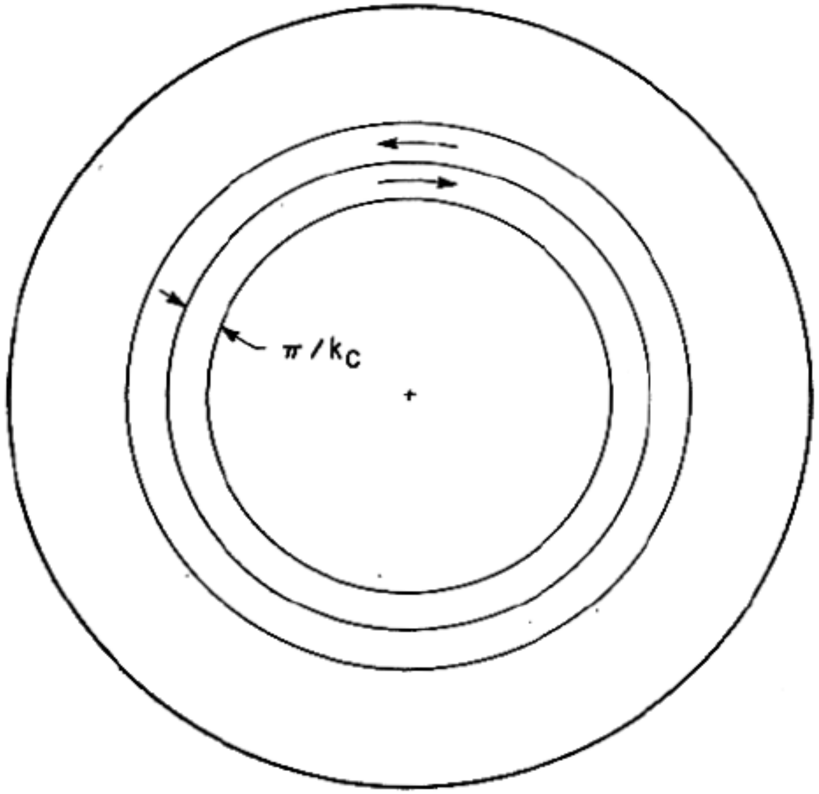
$$and\ M = \frac{\omega_p \left(k_{ry} - k_{qy}\right) + \omega_q \left(k_{py} - k_{ry}\right) + \omega_r \left(k_{qy} - k_{py}\right)}{3\omega_p \omega_q \omega_r}$$

and the wave energy decays conserving the wave quanta,

$$N_k = W_k / \hbar \omega_k$$

And the turbulence spectra shift to lower frequencies and small k_y
=>**Appearance of microscopic zonal flows in the y (azumuthal) directions**. **(Result of the Weak Turbulence Theory).**

Inverse Cascades
-Weak Turbulence Regime



Expected Zonal Flow in the Weak Turbulence Regime
(Hasegawa et al, Phys. Fluids vol.22, p.2122 (1979))

First Observation of Formation of Zonal Flow in a simulation of Strongly Turbulent Toroidal Plasma

(Hasegawa and Wakatani,Phys. Rev. Letters, vol. 59, p.1581 (1987))

Including ion viscosity, μ, electron resistivity, ν_{ei} , and magnetic curvature, $\nabla\Delta$,the equation for the vorticity in a toroidal geometry with minor and major radii, a and R becomes,

$$\frac{\rho_s^2}{a^2}\frac{d}{dt}\nabla^2\phi = \left(\nabla \ln n \times \nabla\Delta\right)\cdot\hat{\mathbf{z}} + \frac{\omega_{ce}}{\nu_{ei}}\left(\frac{a}{R}\right)^2 \nabla_{\shortparallel}^2\left(\ln n - \phi\right) + \frac{\mu}{\omega_{ci}a^2}\nabla_{\perp}^4\phi$$

and the equation of continuity gives,

$$\frac{d}{dt}\ln n = \left(\nabla \ln n \times \nabla\Delta\right)\cdot\hat{\mathbf{z}} + \frac{\omega_{ce}}{\nu_{ei}}\left(\frac{a}{R}\right)^2 \nabla_{\shortparallel}^2\left(\ln n - \phi\right)$$

The instability is driven by the magnetic curvature and pressure gradient.

First Observation of Formation of Zonal Flow in a simulation of Strongly Turbulent Toroidal Plasma

(Hasegawa and Wakatani, Phys. Rev. Letters, vol. 59, p.1581 (1987))

Conservation Laws: In the inviscid limit, here again Energy and Enstrophy are conserved:

1. Energy

$$W = \frac{1}{2}\int\left[\left(\ln n\right)^2 + \frac{\rho_s^2}{a^2}\left(\nabla_\perp \phi\right)^2\right]dV$$

2. Enstrophy

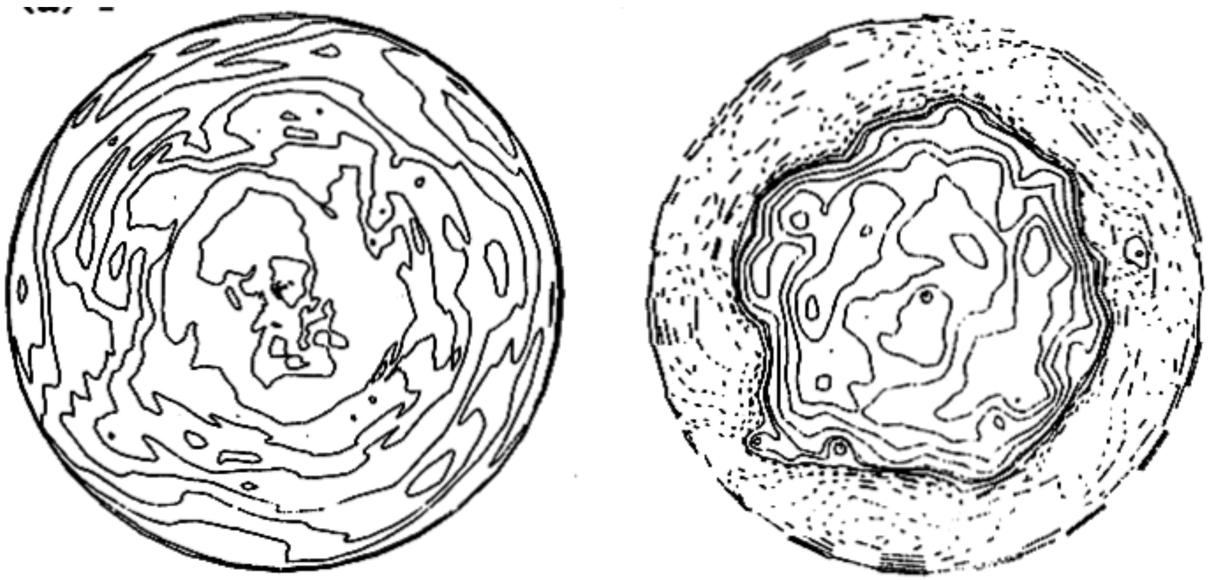
$$U = \frac{1}{2}\int\left[\frac{\rho_s^2}{a^2}\nabla_\perp^2\phi - \ln n\right]^2 dV$$

=>Indication of inverse cascade and formation of (macroscopic) zonal flow: Magnetic curvature drives the instability but does not contribute to the conservation laws

First Observation of Formation of Zonal Flow in a simulation of Strongly Turbulent Toroidal Plasma

(Hasegawa and Wakatani, Phys. Rev. Letters, vol. 59, p.1581 (1987))

Simulation Results

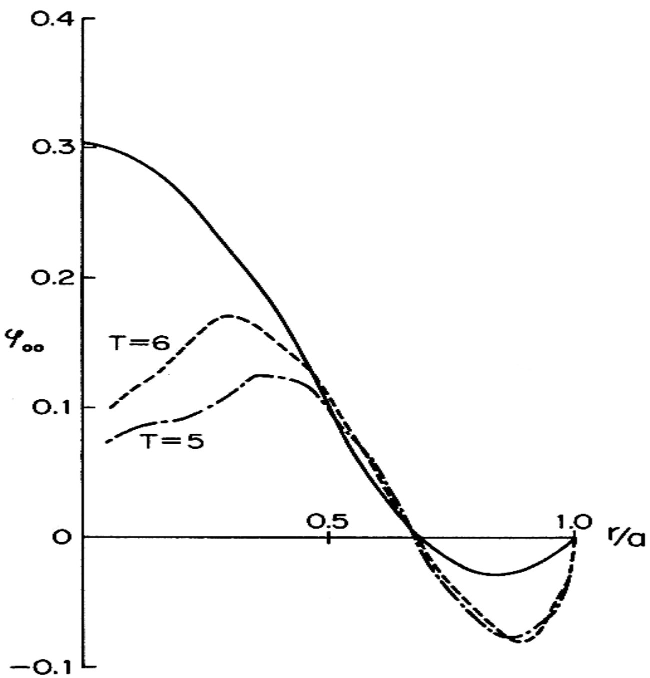


Equipotential lines at initial (left) and final (right) time in the cross section of a cylindrical plasma. Note the appearance of closed equipotential line (stream function), ϕ=0 at the final time.

First Observation of Formation of Zonal Flow in a simulation of Strongly Turbulent Toroidal Plasma

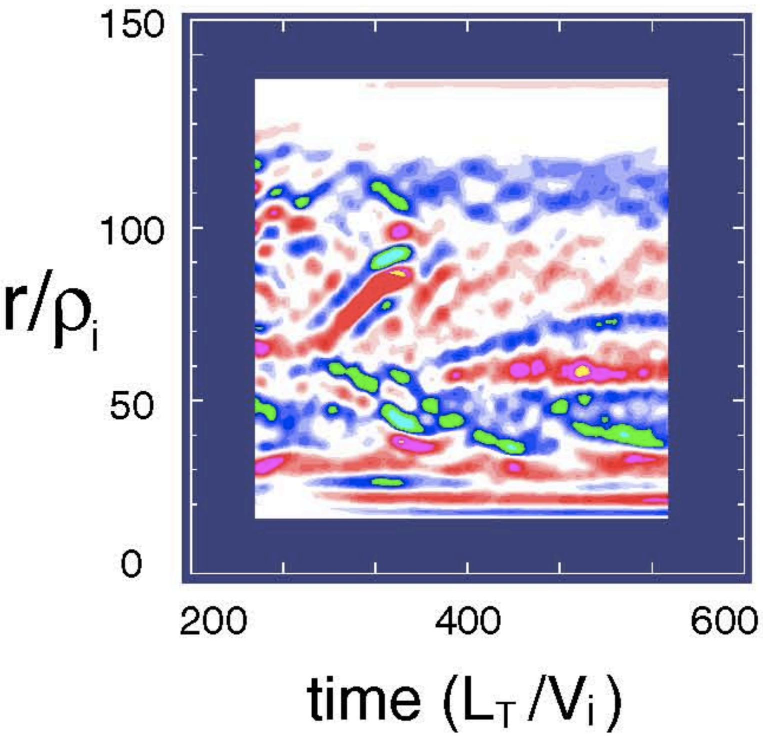
(Hasegawa and Wakatani, Phys. Rev. Letters, vol. 59, p.1581 (1987))

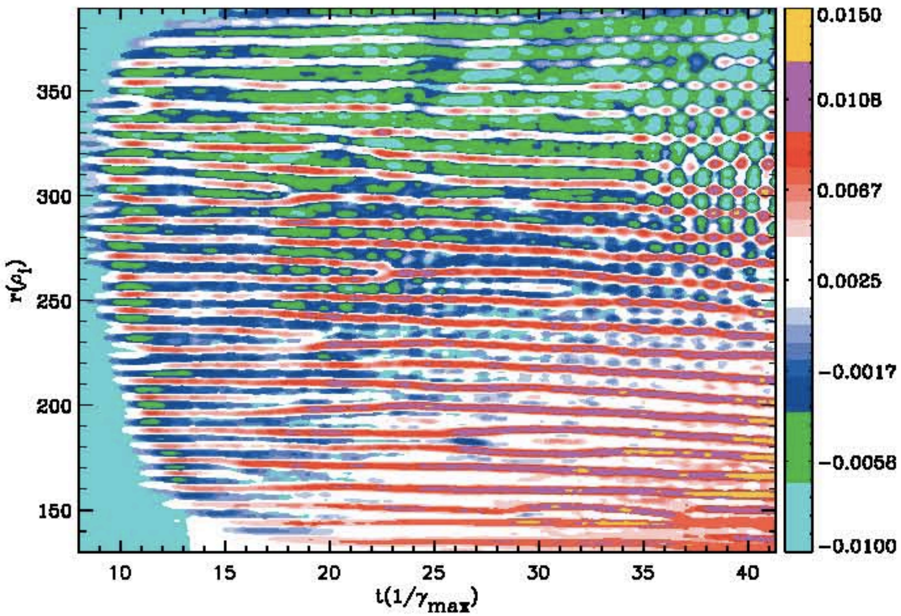
Potential Profile



- The solid line is the theoretical estimate

Zonal Flows observed in Kinetic Codes





ITG Driven Zonal Flow

Hahm et al.Plasma Phy. Cotr.Fusion **42**, A205 (2000)

CTEM Driven Zonal Flow

Xiao et al. Phys. Plasmas **17**, pp. 022302 (2010)

Intermediate Conclusions

General Properties of Electrostatic Turbulence in a Magnetized Cylindrical Plasma.

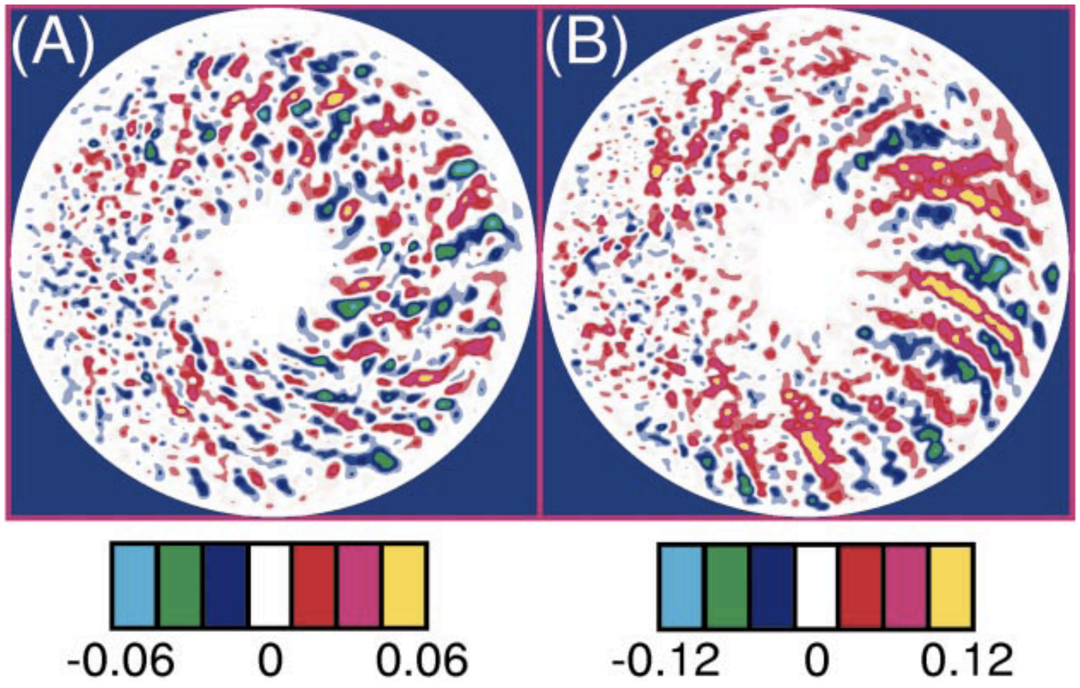
1. Independent of excitation mechanism or nature of instability, zonal flows in the azimuthal direction are formed through turbulent spectrum cascades.
2. In weak to quasi-weak turbulence regime, zonal flows have microscopic structures with radial size that scales like ion gyro-radius.
3. In strong turbulence regime (fully developed turbulence), the zonal flow has a macroscopic structure with radial size a fraction of plasma radius.
4. Radial diffusions of electrons and ions are predominantly determined by the nature of the zonal flow (subject of the second part of the lecture)

Zonal Flow Influence on Particle Transport

1. Zonal flows are expected to inhibit convective motion of eddies across the flow, see the Jovian atmosphere:

Zonal Flow Influence on Particle Transport

2. Effect of zonal flows on Stream Function:
Stream function with (left) and without (right) zonal flows.
Lin et al, Science 281, 1835 (1998).



Zonal Flow Influence on Particle Transport

Since electrons are expected to follow the Boltzmann distribution, electron transports are tied to the stream function (equipotential lines),and zonal flows that distort and inhibit the motion of stream function across them have direct influence on the electron transport.

See the figure below, Xiao et al, Phys.Plasmas **17**, 022302 (2010) of the calculation of the electron heat transport.

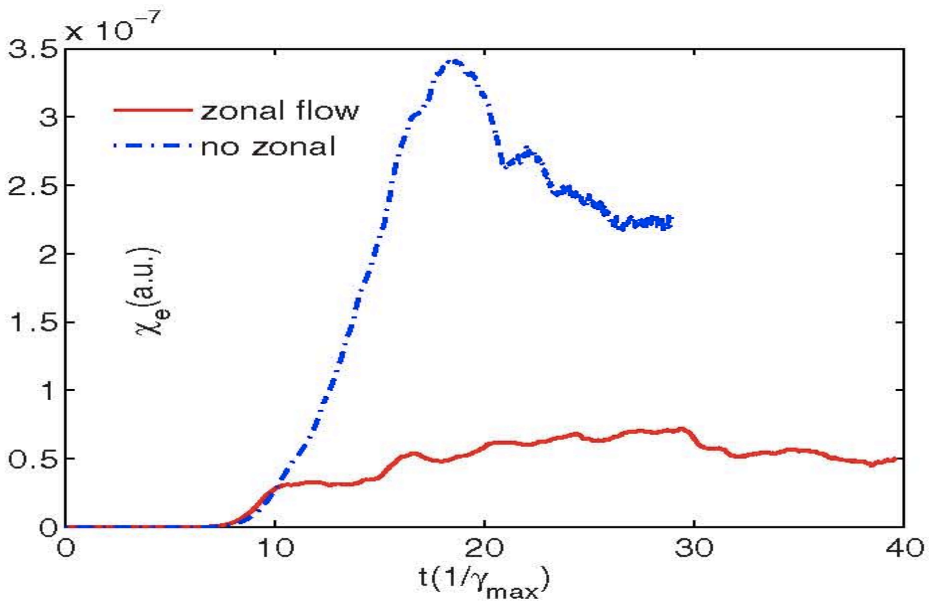


FIG. 2. (Color online) The time history of heat transport for $a/\rho_i = 250$. The solid line has the zonal flow self-consistently generated, while the dotted line has the zonal flow artificially removed.

結言

- 磁場中プラズマの静電乱流を記述するモデル方程式は乱流スペクトルが逆カスケードする性質を表し、結果として円筒プラズマでは円周方向に帯状流を発生する。
- 作られた帯状流は流線を歪め、これに沿って動くプラズマの半径方向の拡散を制限する。
- この結果電子の半径方向の熱伝導は低減される。
- この結果、磁場中プラズマの強い乱流は磁場に垂直方向のプラズマの異常拡散を低減するという興味ある結論が得られる。

謝辞

- 筆者は三月に永眠された指導教授のC. K. Birdsall 博士、また、ソリトンや乱流理論を通じ非線形物理学のご教授を賜った故谷内俊弥先生、及びいくつかの共同研究でお世話になった故若谷誠宏先生に感謝しこの論文を捧げたい。
- また、当講演会を企画頂いた菊池 満先生に感謝する。